Parallel BVH Construction using k-means Clustering

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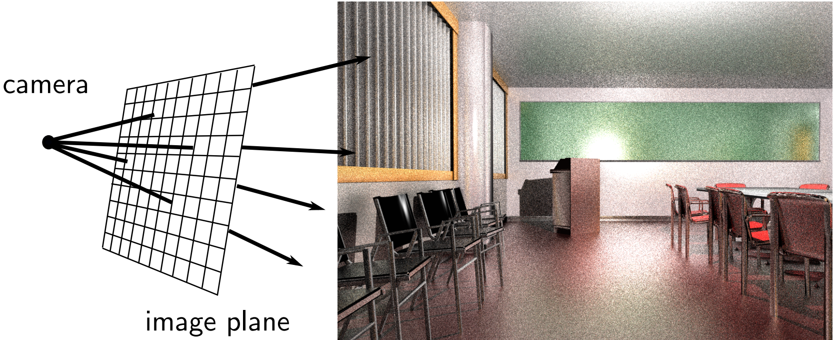
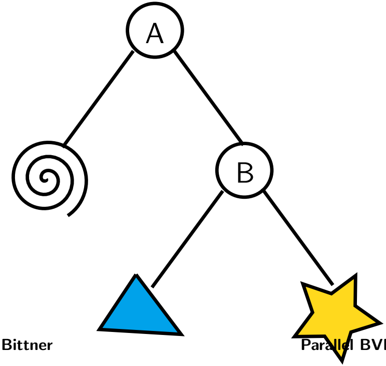
Motivation: Interactive Ray Tracing

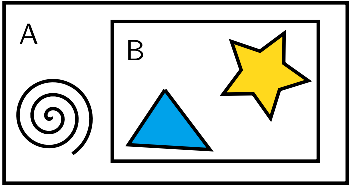
- $t = t_{build} + t_{trace} + t_{shading}$
- Trade-off between BVH quality (t_{trace}) and BVH build (t_{build})
- Goal minimize $t_{build} + t_{trace}$



Bounding Volume Hierarchy

- Ray tracing, collision detection, and visibility culling
- Rooted tree of arbitrary branching factor
  - Geometric primitives in leaves
  - Bounding volumes in interior nodes
- Branching factor 2, axis aligned bounding boxes





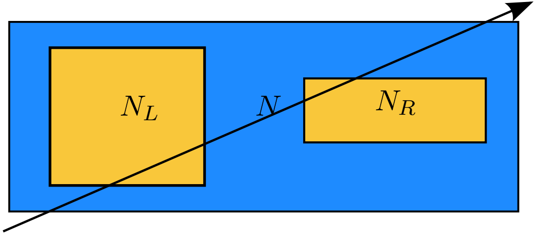
[Clark 1976]

Surface Area Heuristic

$(2n-3)!!$[1] valid binary bounding volume hierarchies

$$c(N) = \begin{cases} k_T + P(N_L|N)c(N_L) + P(N_R|N)c(N_R) & \text{if } N \text{ is interior node} \\ k_I|N| & \text{otherwise} \end{cases}$$

$$P(N_L|N) = \frac{SA(N_L)}{SA(N)} \qquad P(N_R|N) = \frac{SA(N_R)}{SA(N)}$$



[MacDonald and Booth 1990]

[1] double factorial defined as $k!! = k(k-2)(k-4)\dots 1$ for odd k

BVH Construction Methods

Top-down

- Surface Area Heuristic [Hunt et al. 2007, ...]
- Binning [Wald 2007, ...]

Bottom-up

- Agglomerative clustering [Walter et al. 2008, Gu et al. 2013]

Insertion

- Heuristic greedy search [Goldsmith and Salmon 1987]
- Online construction [Bittner et al. 2015]

BVH Construction Methods

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- **k-means clustering**

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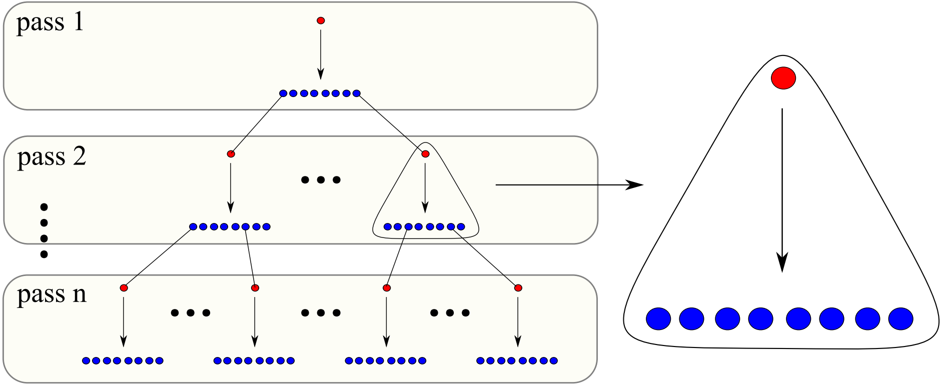
k-means Clustering



k-means Clustering

Lloyd's algorithm [Lloyd 1982]

- Initialize k representatives
- k-means loop
  - Assignment step
  - Update step

Initialization of k representatives $\mathbf{r}_1, \ldots, \mathbf{r}_k$

- Randomly draw $\mathbf{r}_1$
- To select $\mathbf{r}_i$
  - Randomly draw p candidates
  - Select one maximizing distance to the nearest already selected representative

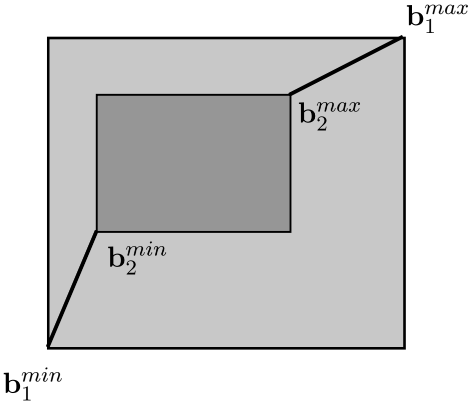
BVH Construction via k-means Clustering

Distance d between bounding boxes $\mathbf{b}_1$ and $\mathbf{b}_2$

$$d(\mathbf{b}_1, \mathbf{b}_2) = \|\mathbf{b}_1^{min} - \mathbf{b}_2^{min}\|^2 + \|\mathbf{b}_1^{max} - \mathbf{b}_2^{max}\|^2$$

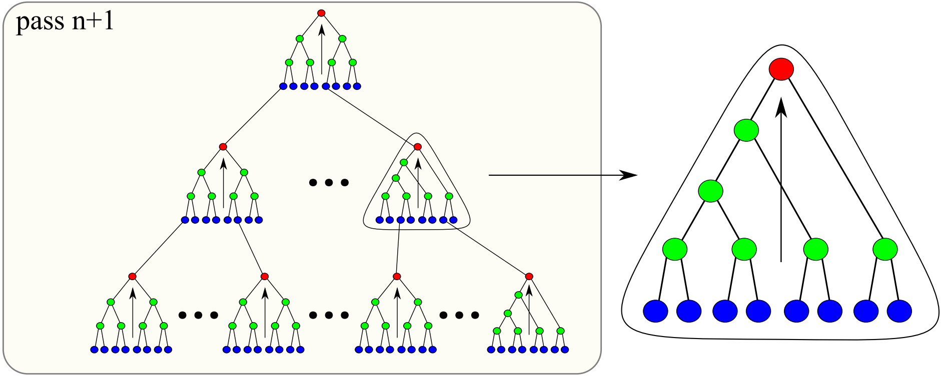
Positions of representatives

$$\mathbf{r}_i^{t+1} = \frac{1}{|\mathcal{C}_i^t|}\Big(\sum_{\mathbf{b}_j \in \mathcal{C}_i^t} \mathbf{b}_j^{min}, \sum_{\mathbf{b}_j \in \mathcal{C}_i^t} \mathbf{b}_j^{max}\Big)$$



Agglomerative Clustering

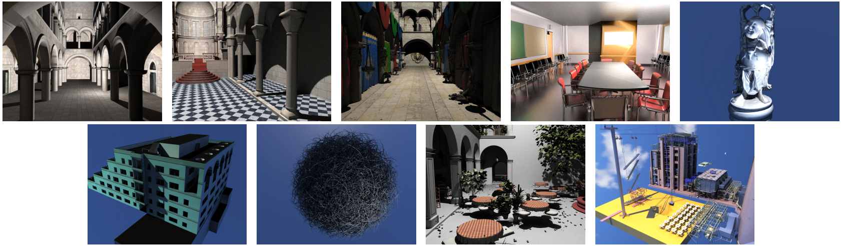
- All treelets processed in parallel
- Naïve algorithm without additional data structures $\mathcal{O}(n^3)$
- Surface area of merged cluster as a distance between clusters [Walter et al. 2008]



k-means Visualization

Results

- 9 scenes, 66k - 12M tris, 3 representatives views, resolution 1024×768
- Path tracing with Russian roulette (GPU ray tracing kernel [Aila and Laine 2009])
  - Low quality rendering (total time LQ): 8 samples per pixel, 2 shadow samples
  - High quality rendering (total time HQ): 128 samples per pixel, 2 shadow samples
- Intel Core I7-3770 3.4 GHz CPU (4 cores), 16 GB RAM
- CUDA: NVIDIA GeForce GTX TITAN Black, 6 GB RAM



Tested Methods

parameters

- k ... the number of clusters
- p ... the number of candidates in initialization procedure
- i ... the number of iterations

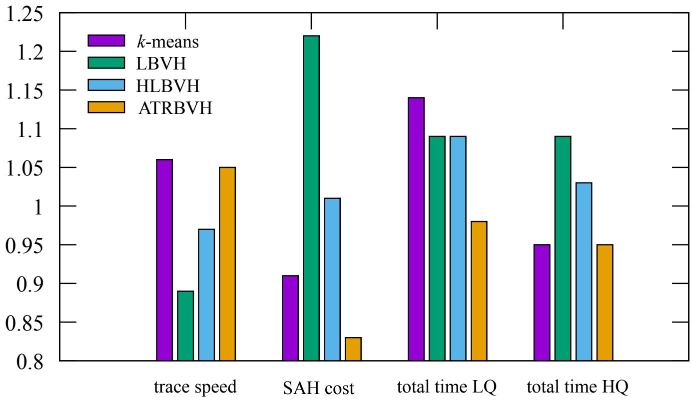
reference methods

- Full sweep SAH (idealized with zero build time)
- LBVH [Karras 2012]: 60-bit Morton codes
- HLBVH [Garanzha et al. 2011]: 60-bit Morton codes, 15 bits for SAH splits
- ATRBVH [Domingues and Pedrini 2015]: 2 iterations, treelet size 9

Max. 8 triangles per leaf for all methods, SAH cost constants $k_T = 3$, $k_I = 2$

Sponza

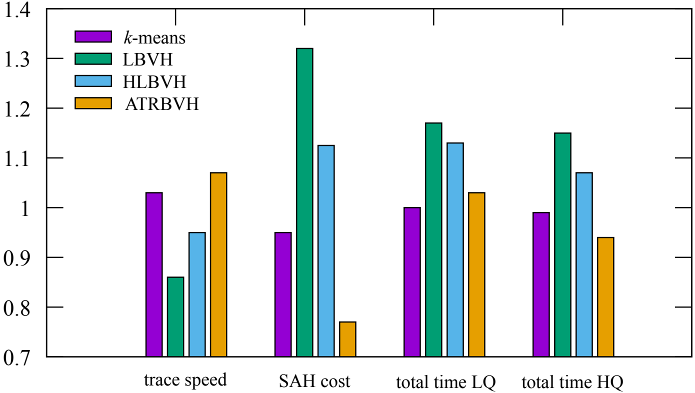
66k tris, $k = 32$, $p = 20$, $i = 10$



Normalized with the respect to idelized full sweep SAH builder

Conference

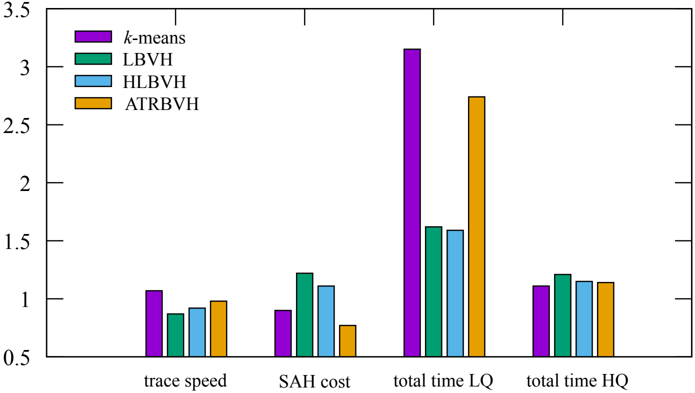
331k tris, $k = 8,\ p = 5,\ i = 2$

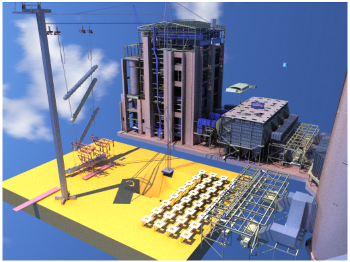


Normalized with the respect to idelized full sweep SAH builder

Power Plant

12759k tris, $k = 16,\ p = 5,\ i = 5$



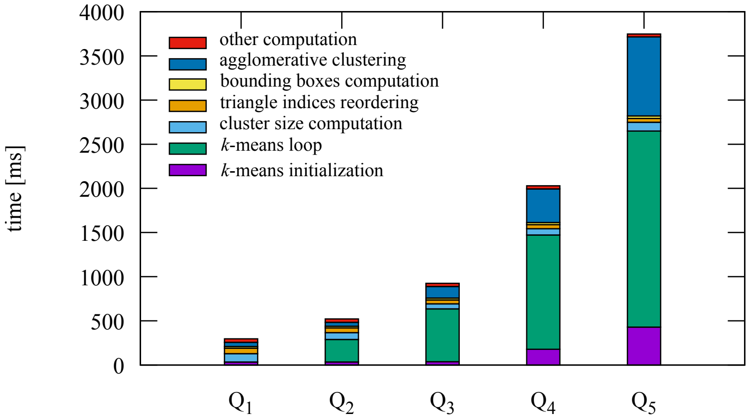


Normalized with the respect to idelized full sweep SAH builder

Power Plant

Kernel times of BVH construction

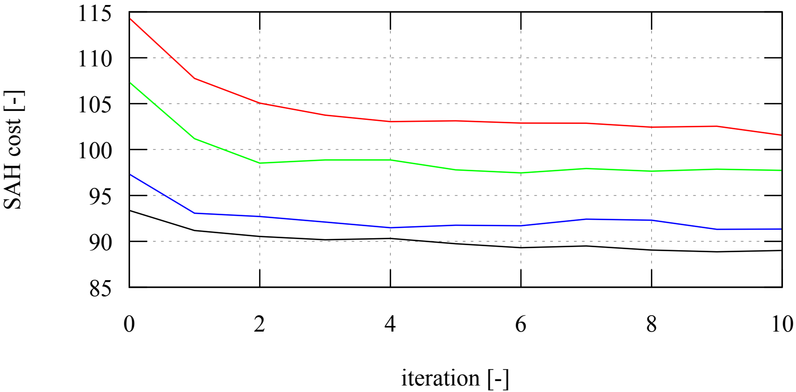
- Q_1: $k = 8,\ p = 5,\ i = 0$
- Q_2: $k = 8,\ p = 5,\ i = 2$
- Q_3: $k = 16,\ p = 5,\ i = 5$
- Q_4: $k = 32,\ p = 20,\ i = 10$
- Q_5: $k = 64,\ p = 30,\ i = 15$



Power Plant

SAH cost stabilization rate ($k_T = 3$, $k_I = 2$)

- $k = 8$, $p = 5$ (red)
- $k = 16$, $p = 5$ (green)
- $k = 32$, $p = 20$ (blue)
- $k = 64$, $p = 30$ (black)



Conclusion and Future Work

BVH construction

- k-means and agglomerative clustering
- Viable alternative to Morton ordering
- Implementation in CUDA

Future work

- k-ary BVH
- Evaluation for larger k
- Other distance functions (e.g. k-medians)

Thank you for your attention!

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Euclidean Distance

$$\min_{\mathbf{r}} \sum_{i=1}^{n} \|\mathbf{x}_i - \mathbf{r}\|_2^2$$

$$\frac{\mathbf{d}}{\mathbf{dr}} \left(\sum_{i=1}^{n} \|\mathbf{x}_i - \mathbf{r}\|_2^2 \right) = -2 \sum_{i=1}^{n} (\mathbf{x}_i - \mathbf{r})$$

$$0 = -2 \sum_{i=1}^{n} (\mathbf{x}_i - \mathbf{r}) = \sum_{i=1}^{n} (\mathbf{x}_i - \mathbf{r}) = \sum_{i=1}^{n} \mathbf{x}_i - n\mathbf{r}$$

$$\mathbf{r} = \frac{1}{n} \sum_{i=1}^{n} \mathbf{x}_i \text{ (mean)}$$

Manhattan Distance

$$\min_{\mathbf{r}} \sum_{i=1}^{n} \|\mathbf{x}_i - \mathbf{r}\|_1$$

$$\frac{\mathbf{d}}{\mathbf{dr}}\left(\sum_{i=1}^{n} \|\mathbf{x}_i - \mathbf{r}\|_1\right) = \sum_{i=1}^{n} \frac{\mathbf{d}}{\mathbf{dr}} \|\mathbf{x}_i - \mathbf{r}\|_1 = \sum_{i=1}^{n} \sum_{j=1}^{d} \frac{\mathbf{d}}{\mathbf{dr}^j} |\mathbf{x}_i^j - \mathbf{r}^j| = 0$$

$$\frac{\mathbf{d}}{\mathbf{dr}^j} |\mathbf{x}_i^j - \mathbf{r}^j| = \begin{cases} -1 & \mathbf{x}_i^j > \mathbf{r}^j \\ 1 & \mathbf{x}_i^j < \mathbf{r}^j \end{cases}$$

$$\mathbf{r}^j = \begin{cases} \mathbf{x}^j_{\left(\frac{n+1}{2}\right)} & n \text{ odd} \\ \frac{1}{2}\left(\mathbf{x}^j_{\left(\frac{n}{2}\right)} + \mathbf{x}^j_{\left(\frac{n}{2}+1\right)}\right) & n \text{ even} \end{cases} \quad \text{(component-wise median)}$$