Digital Image

(B4M33DZO, Winter 2024)

Lecture 1:

Monadic Operations

https://cw.fel.cvut.cz/wiki/courses/b4m33dzo/start

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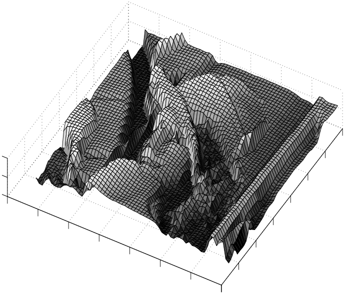
What is the image?

In theory – continuous function of two variables:

$$I(x, y) : \mathbf{R}^2 \Rightarrow \mathbf{R}^n$$

where $n = 1$: luminance, $n = 3$: color, etc.



In practice – matrix of pixels (equidistant samples):

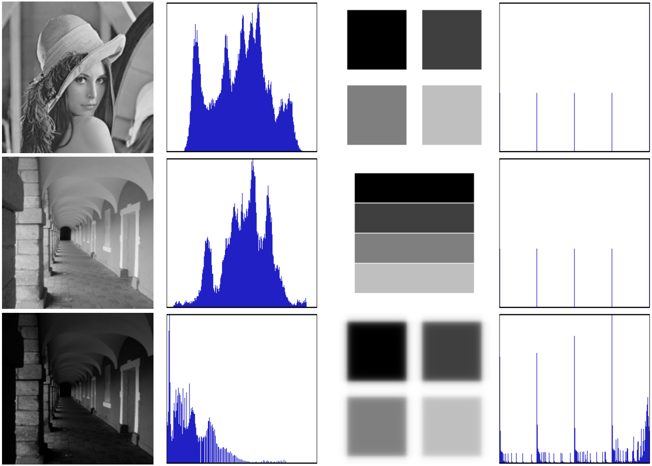
$$I[x, y] : \mathbf{N}^2 \Rightarrow \mathbf{R}^n$$

What is image processing?

Specific modification of image function:

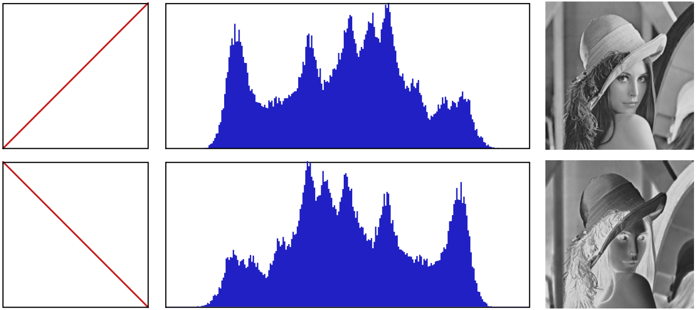
Histogram

Probability density function (pdf) of pixel luminance:



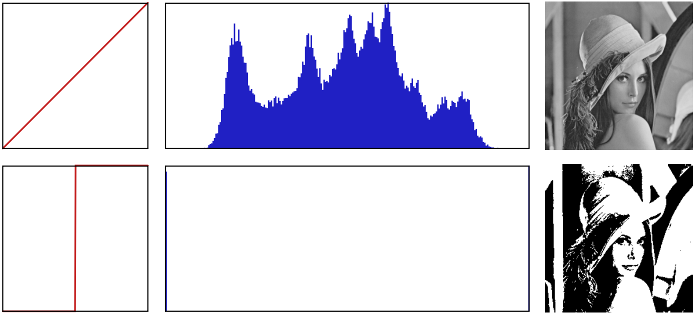
Monadic Operations

Negative: $T(l) = 1 - l$



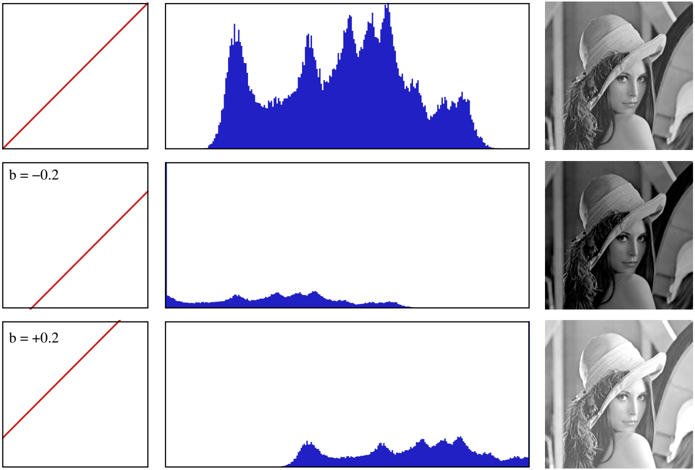
Monadic Operations

Threshold: $T(l) = \begin{cases} 0 & l < \theta \\ 1 & l \geq \theta \end{cases}$



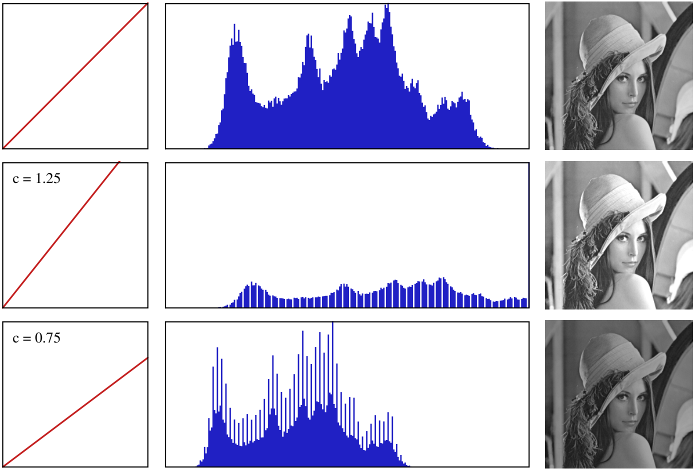
Monadic Operations

Brightness: $T(l) = l + b \quad b \in \langle -1, 1 \rangle$



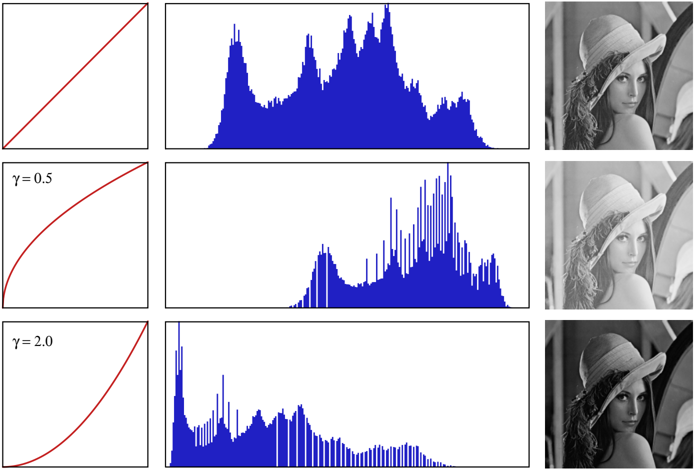
Monadic Operations

Contrast: $T(l) = l \cdot c \quad c \in \langle 0, \infty)$



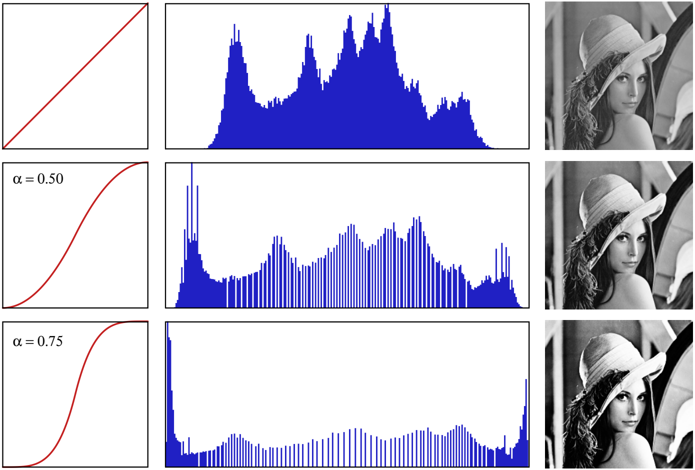
Monadic Operations

Gamma correction: $T(l) = l^{\gamma} \quad \gamma \in (0, \infty)$



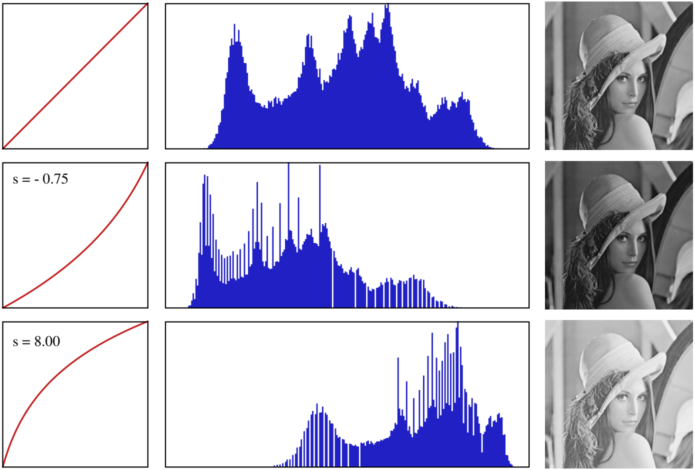
Monadic Operations

Non-linear contrast: $T(l) = \begin{cases} \frac{1}{2}(2l)^\gamma & l \in \langle 0, \frac{1}{2}) \\ 1 - \frac{1}{2}(2-2l)^\gamma & l \in \langle \frac{1}{2}, 1 \rangle \end{cases} \quad \gamma = \frac{1}{1-\alpha} \quad \alpha \in (0,1)$



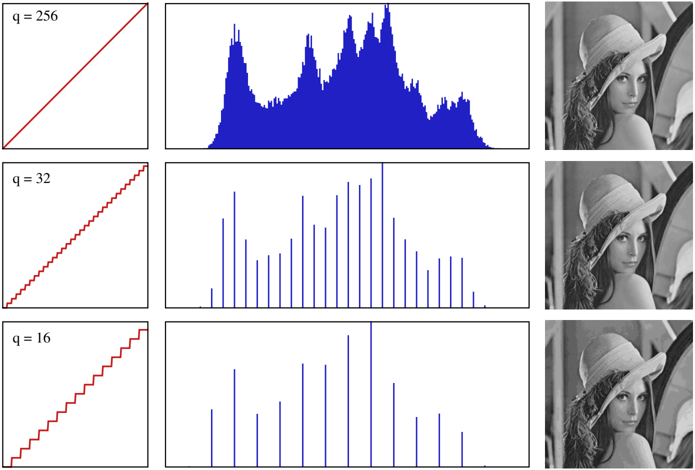
Monadic Operations

Logarithmic scale: $T(l) = \frac{\log(1+l \cdot s)}{\log(1+s)} \quad s \in (-1, \infty)$



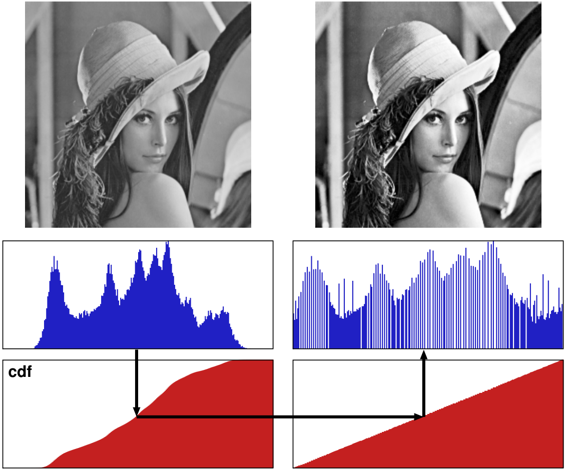
Monadic Operations

Quantization: $T(l) = \lfloor l \cdot q \rfloor \quad q \in (1, \infty)$



Monadic Operations

Equalization: $T(l) = \mathrm{cdf}(l) = \sum_{i=0}^{l} \mathrm{pdf}(i)$



Monadic Operations

Change of variable: $u = T(l)$

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$$\text{pdf}_U(u) = \text{pdf}_L(T^{-1}(u)) \left| \frac{d}{du} T^{-1}(u) \right|$$

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Monadic Operations

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$$\frac{du}{dl} = \frac{d}{dl} T(l) \quad \Rightarrow \quad \frac{dl}{du} = \frac{1}{\frac{d}{dl} T(l)}$$

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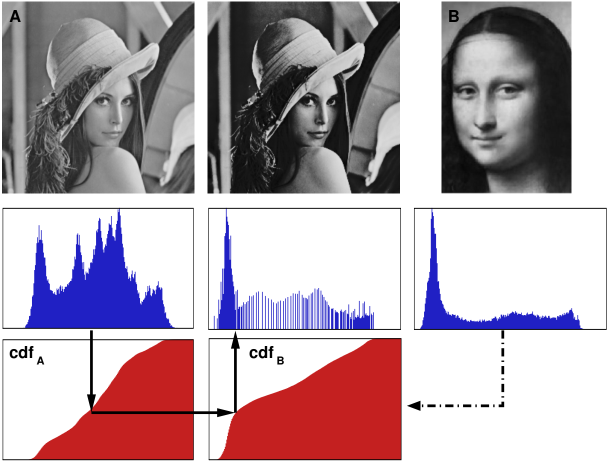
$$\mathrm{pdf}_L(l) = \frac{d}{dl} \mathrm{cdf}_L(l)$$

$$\mathrm{pdf}_U(u) = \mathrm{pdf}_L(l) \left| \frac{1}{\mathrm{pdf}_L(l)} \right| \quad \Rightarrow \quad \mathrm{pdf}_U(u) = 1$$

Monadic Operations

Mapping: $T(l) = \mathrm{cdf}_B^{-1}(\mathrm{cdf}_A(l))$



Monadic Operations

Composite: $v = S(T(l))$

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$$\mathrm{pdf}_A(l) \quad \xrightarrow{T} \quad \mathrm{pdf}_U(u) \quad \xrightarrow{S} \quad \mathrm{pdf}_B(v)$$

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$$T(l) = \mathrm{cdf}_A(l) \qquad \mathrm{pdf}_U(u) = 1 \qquad S(u) = \mathrm{cdf}_B^{-1}(u)$$

Monadic Operations

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$$\mathrm{pdf}_B(v) = \mathrm{pdf}_U(u) \left| \frac{du}{dv} \right| = 1 \cdot \left| \frac{du}{dv} \right| = \left| \frac{d}{dv} \mathrm{cdf}_B(v) \right|$$

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$$v = \mathrm{cdf}_B^{-1}(u) \longrightarrow u = \mathrm{cdf}_B(v)$$

$$\mathrm{pdf}_B(v) = \mathrm{pdf}_U(u)\left|\frac{du}{dv}\right| = 1 \cdot \left|\frac{du}{dv}\right| = \left|\frac{d}{dv}\mathrm{cdf}_B(v)\right|$$

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