Digital Image

(B4M33DZO, Winter 2024)

Lecture 4:

Linear Filtering

https://cw.fel.cvut.cz/wiki/courses/b4m33dzo/start

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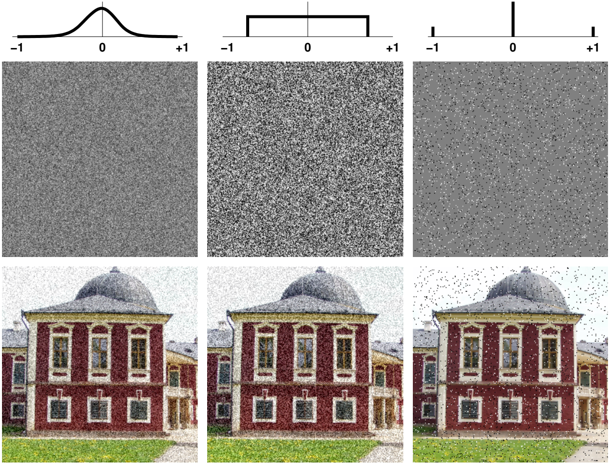
Noise Suppression

Model of additive noise: $\color{red}{H} = \color{blue}{F} + \color{green}{N}$

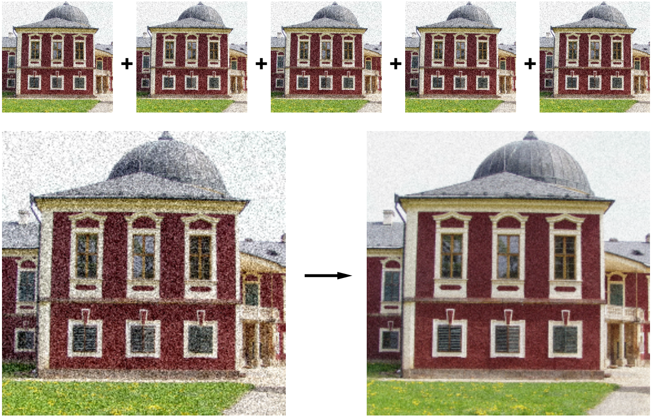


gaussian **uniform** **impulsive**

Noise Suppression

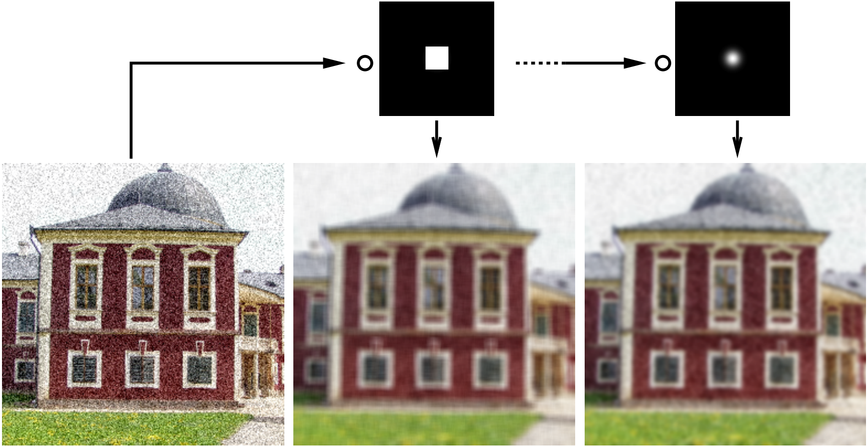
Average of k images (degraded by Gaussian noise $\mathrm{N} \sim \mathcal{N}\left(0, \sigma^2\right)$):

$$\mathrm{pdf}\left\{\frac{1}{k}\sum_{i=1}^{k} n_i\right\} = \mathcal{N}\left(0, \frac{\sigma^2}{k}\right)$$



Noise Suppression

Weighted average of neighbor pixels (Box & Gaussian kernel):

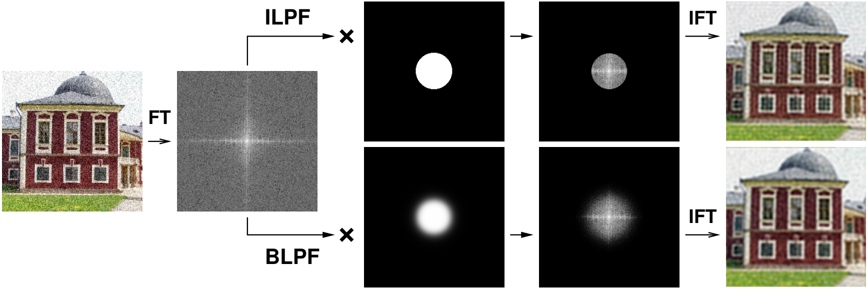


noise | average | Gaussian blur

Noise Suppression

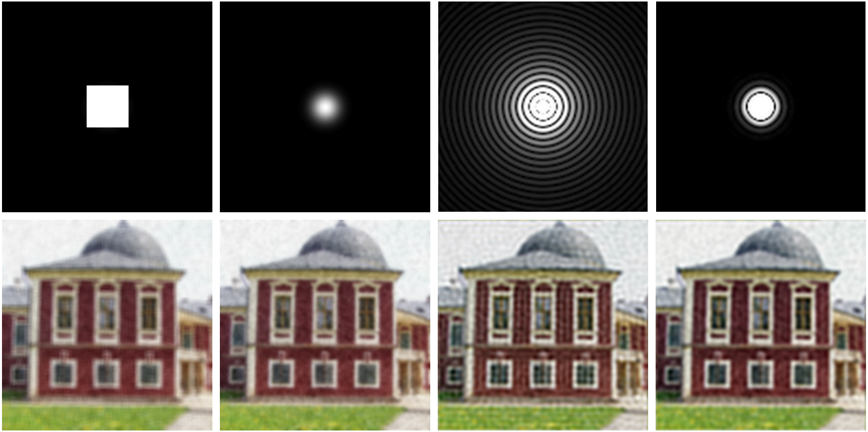
Low-pass filtering (ideal low-pass filter & Butterworth filter):

$$\mathbf{ILPF}(u,v) = \begin{cases} 1, & \sqrt{u^2+v^2} < \sigma; \\ 0, & \text{elsewhere.} \end{cases} \qquad \mathbf{BLPF}(u,v) = \frac{1}{1+\left(\sqrt{u^2+v^2}/\sigma\right)^{2\gamma}}$$



Noise Suppression

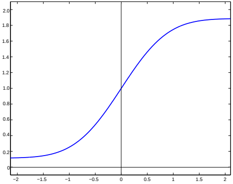
Box	Gauss	ILPF	BLPF



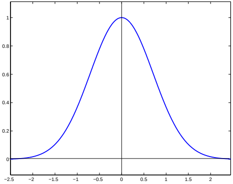
Edge Detection

Model of edge (in 1D):

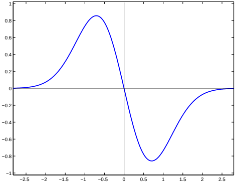
$f(x)$



$f'(x)$



$f''(x)$

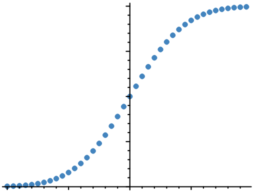


edge position: $\arg\max_x f'(x)$ $\quad$ $f''(x) = 0$

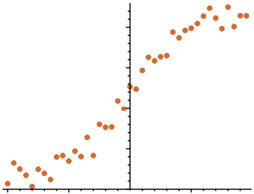
Edge Detection

Discrete case with noise:

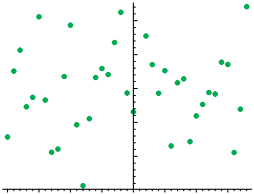
$\mathrm{f}[x]$



$\mathrm{h}[x] = \mathrm{f}[x] + \mathrm{n}[x]$



$\mathrm{h}'[x] = \mathrm{h}[x] - \mathrm{h}[x-1]$



Problem: how to estimate f' when f is corrupted by noise n?

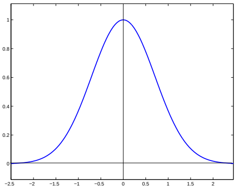
Edge Detection

Solution: suppress noise by convolution with g and differentiate.

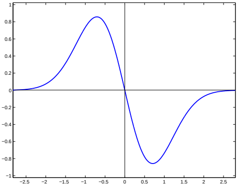
Differentiation can be computed analytically (in advance):

$$(f * g)' = f * g' \quad \Rightarrow \quad \sum_x \mathrm{f}[x] * \mathrm{g}'[t-x]$$

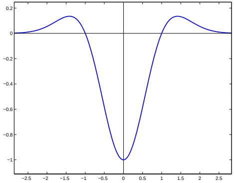
$$(f * g)'' = f * g'' \quad \Rightarrow \quad \sum_x \mathrm{f}[x] * \mathrm{g}''[t-x]$$



$g(x)$

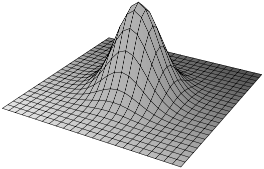


$g'(x)$

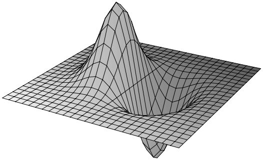


$g''(x)$

Edge Detection

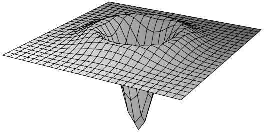


$$G(x,y) \propto \exp\left(-\frac{x^2+y^2}{\sigma^2}\right)$$



$$G'_x(x,y) \propto \frac{x}{\sigma^2}\exp\left(-\frac{x^2+y^2}{\sigma^2}\right)$$

$$\sqrt{(F * G'_x)^2 + (F * G'_y)^2} \quad \arctan\left(\frac{F * G'_y}{F * G'_x}\right)$$



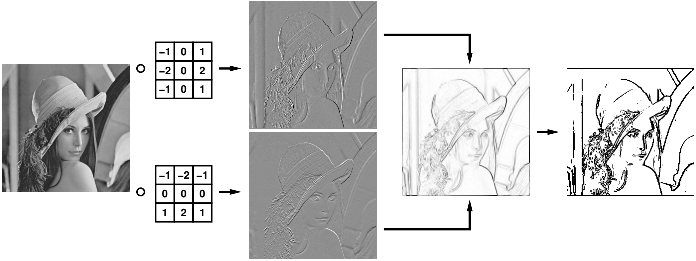
$$\nabla^2 G(x,y) \propto \frac{x^2+y^2-\sigma^2}{\sigma^4}\exp\left(-\frac{x^2+y^2}{\sigma^2}\right)$$

$$\nabla^2 G(x,y) = g''(x)\cdot g(y) + g(x)\cdot g''(y)$$

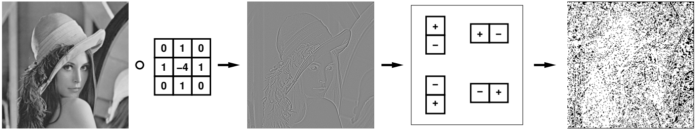
Edge Detection

Simple approximations:

Sobel edge detector:



Laplacian edge detector:

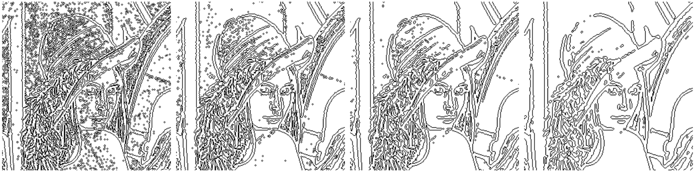


Edge Detection

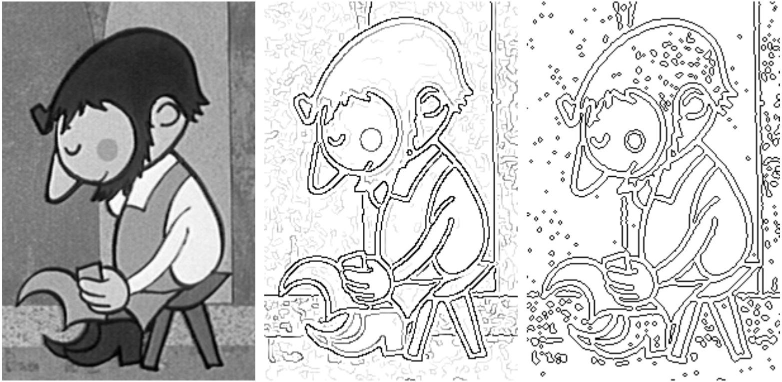
Varying σ (scale-space):

Canny edge detector (non-maxima suppression of $F * G'_\alpha$):

Laplacian of Gaussian zero-crossings ($F * \nabla^2 G = 0$):



Outline detection and segmentaion:

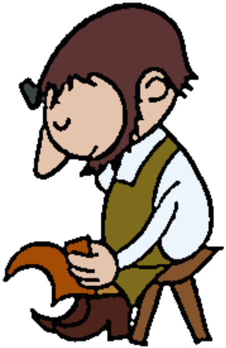


$\max F * G'_{\alpha}$ $F * \nabla^2 G = 0$

Outline detection and segmentaion:

flood-fill

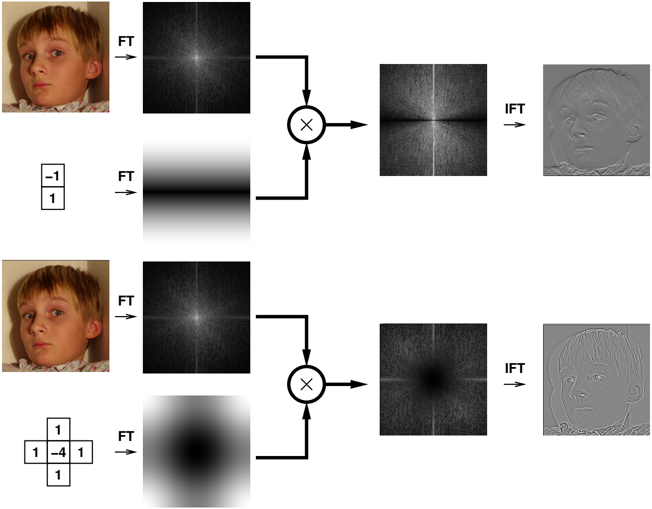
erosion



segmentation

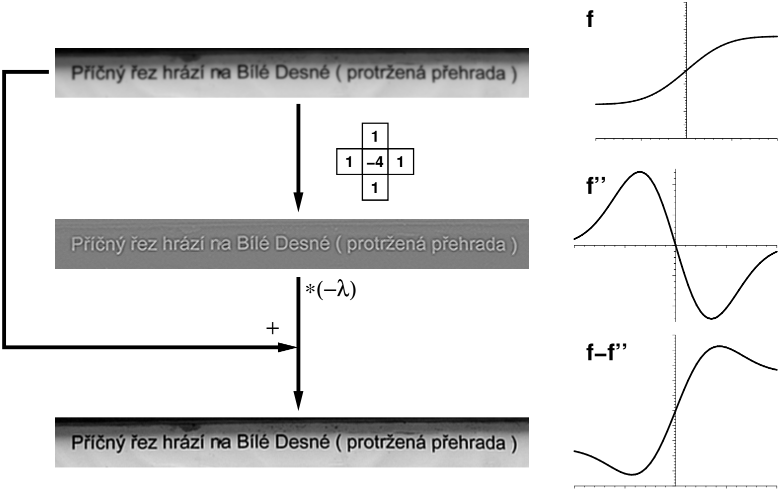
Edge Detection

Hi-pass filtering (approximation of the 1st and 2nd derivatives):

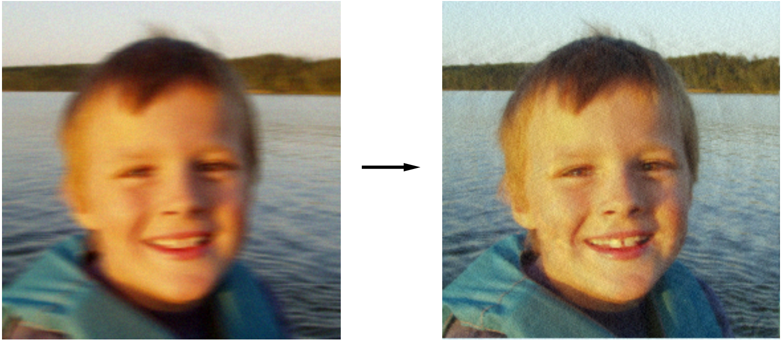


Edge Detection

Sharpening (locally increase contrast of edges):

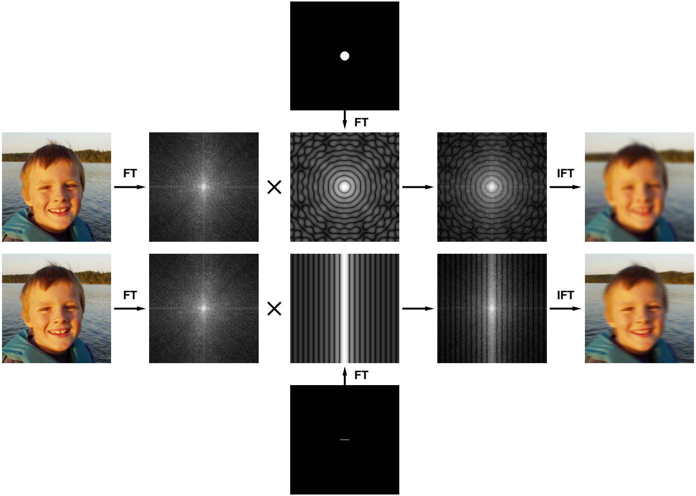


Deblurring



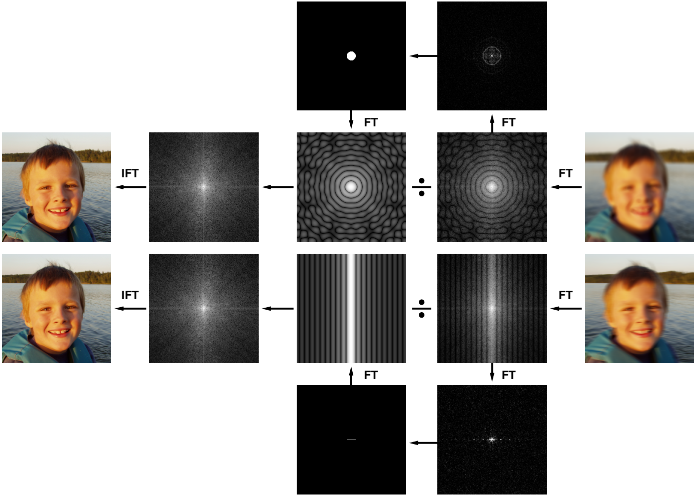
Deblurring

Modelling out-of-focus & motion blur in Fourier domain:



Deblurring

Recover image from degraded observation (deconvolution):



Wiener Filtration

Problem: image f is also degraded by unknown additive noise n.

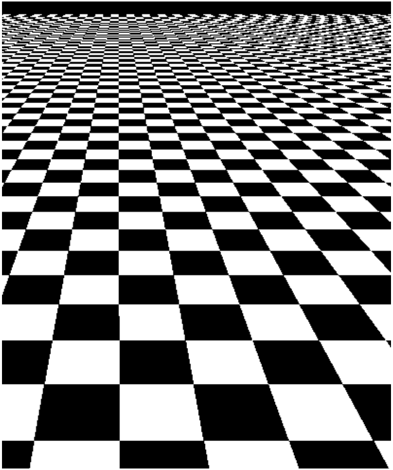
$$g = h * f + n \quad \iff \quad G = H \cdot F + N \quad \Rightarrow \quad F = (G - N)/H$$

Solution: find an image $\hat{f}$ such that $||f - \hat{f}||^2$ is minimal.

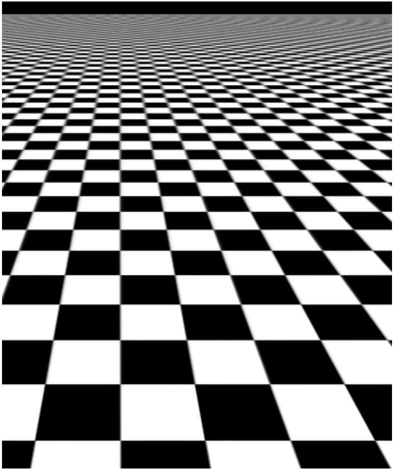
$$\hat{F}(u,v) = \frac{H^*(u,v) \cdot G(u,v)}{||H(u,v)||^2 + \lambda} \qquad \lambda = \mathrm{SNR}^{-1} = \frac{||N(u,v)||^2}{||F(u,v)||^2}$$

Anti-Aliasing

nearest neighbor

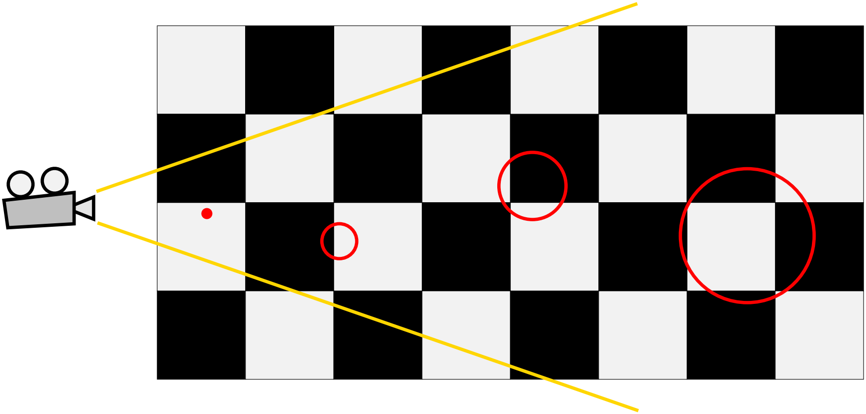


filtered



Anti-Aliasing

Kernel size with increasing distance from camera:



Depth-of-Field

large

small

Depth-of-field

Projection of in-focus and out-of-focus points:

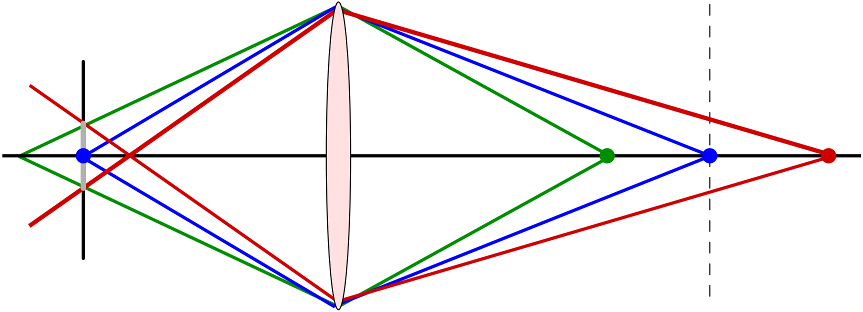
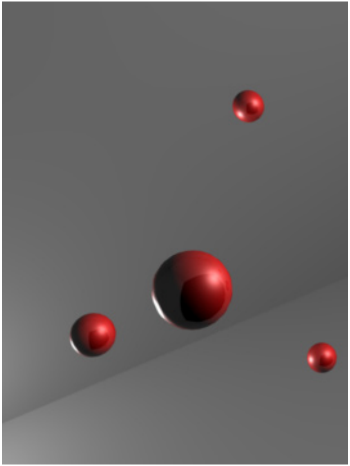


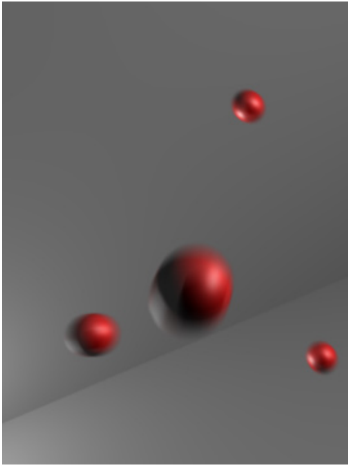
Image of out-of-focus point is circle (circle of confusion).

Motion Blur

without motion blur

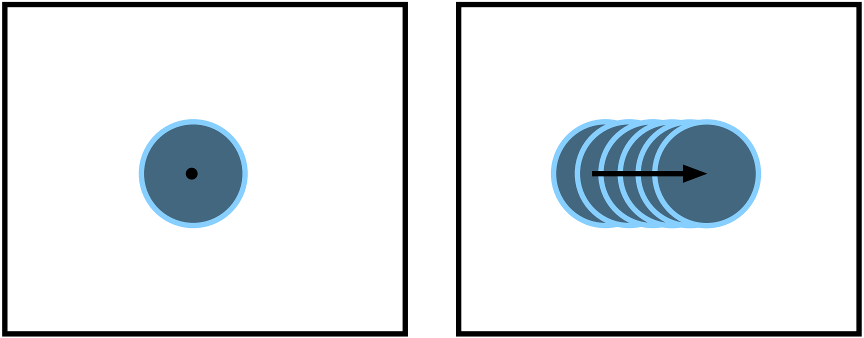


with motion blur



Motion Blur

Object is exposed continuously along its motion path:

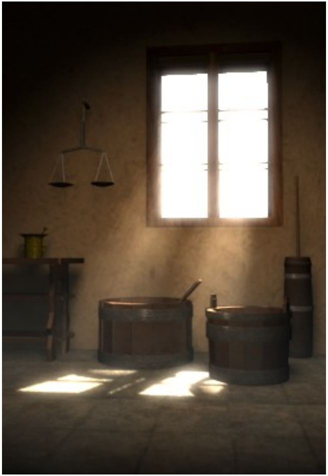


Motion blur is temporal analogy to spatial anti-aliasing.

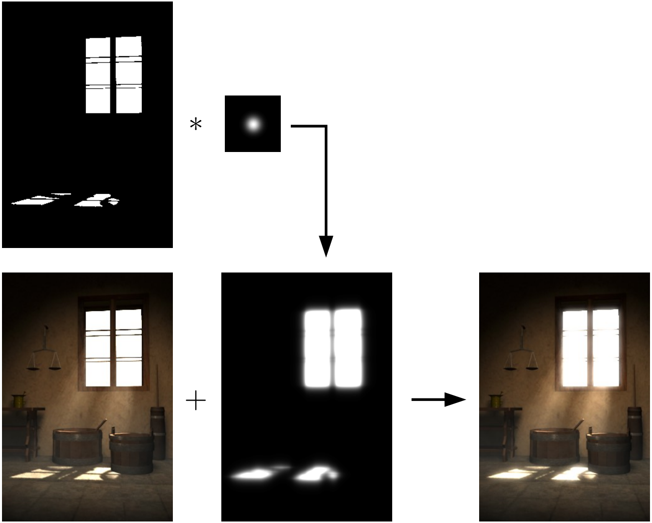
Diffraction Glow

without glow

with glow

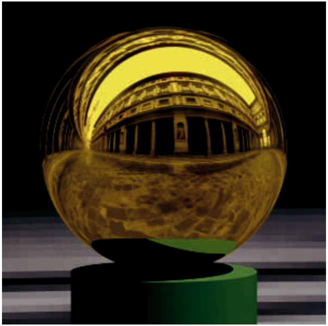


Diffraction Glow

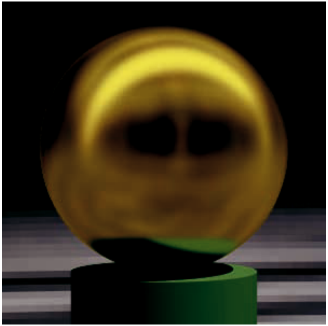


Glossy Reflection

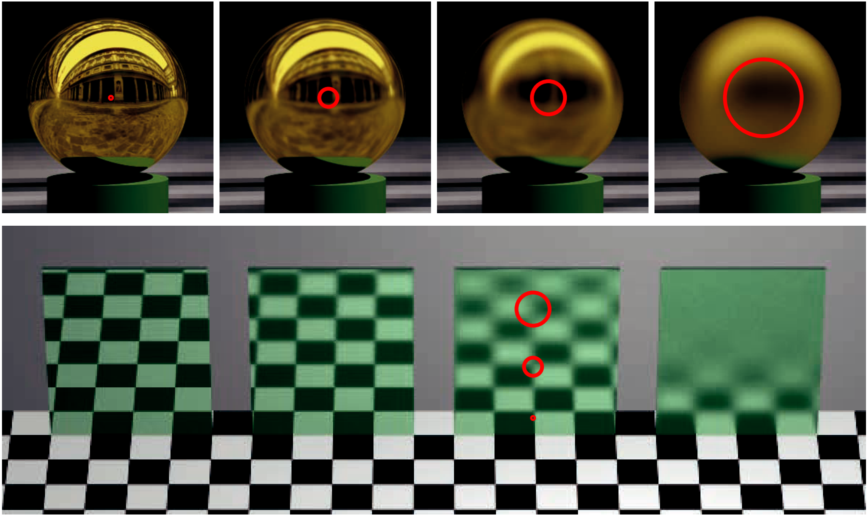
high-frequency BRDF



low-frequency BRDF



Glossy Reflection



Soft Shadows

hard shadows

soft shadows

Hybrid Images

Hybrid Images

Hybrid Images