

Digital Image

(B4M33DZO, Winter 2024)

Lecture 6:

Image Editing

<https://cw.fel.cvut.cz/wiki/courses/b4m33dzo/start>

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How to stitch two similar images?



We can use histogram mapping to reach similar intensity level.

Problem: local changes still remain visible along the seam.

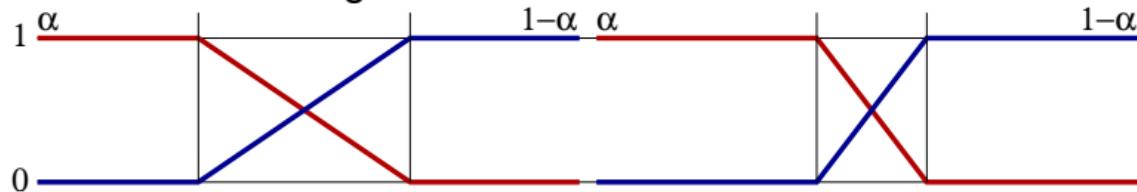
Spatially weighted linear combination of images:

$$\mathbf{I}[x, y] = \alpha \cdot \mathbf{I}_a[x, y] + (1 - \alpha) \cdot \mathbf{I}_b[x, y]$$

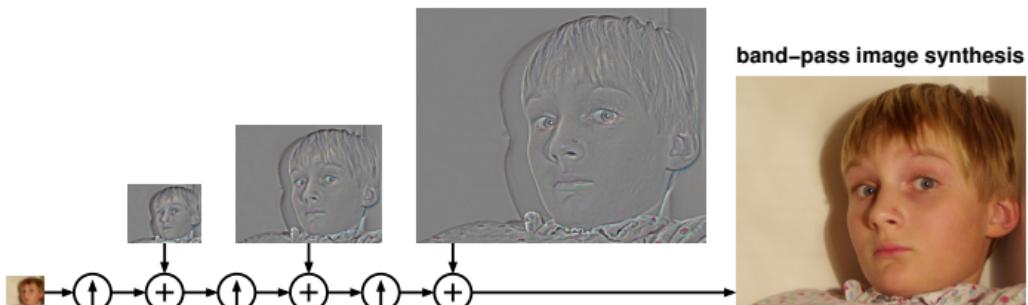
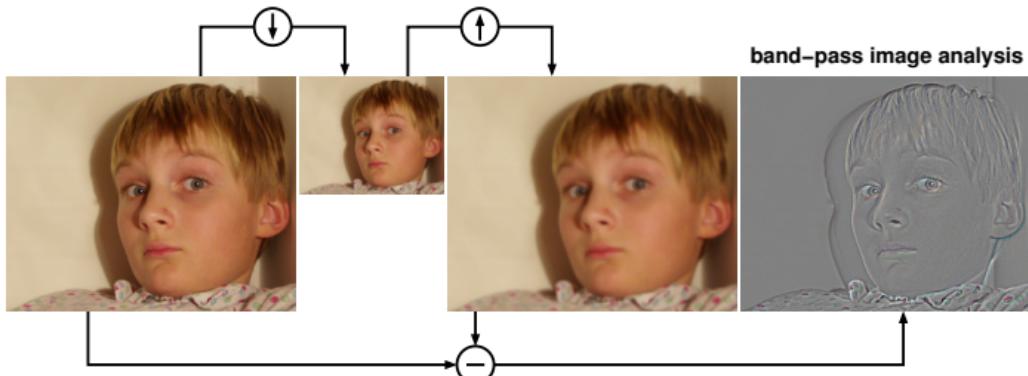


too wide – ghosts

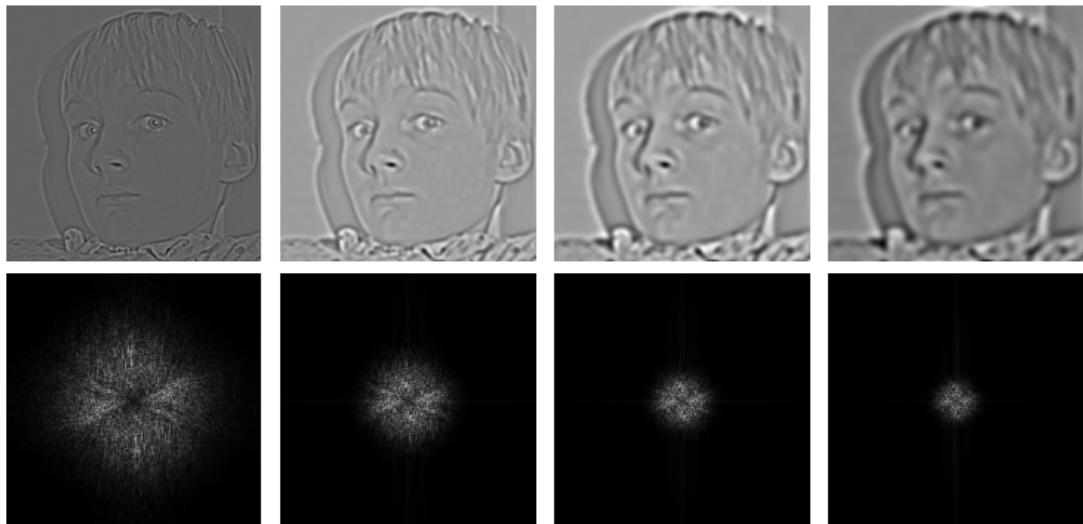
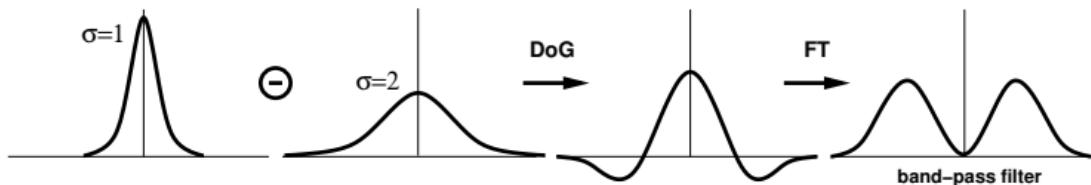
too narrow – seam



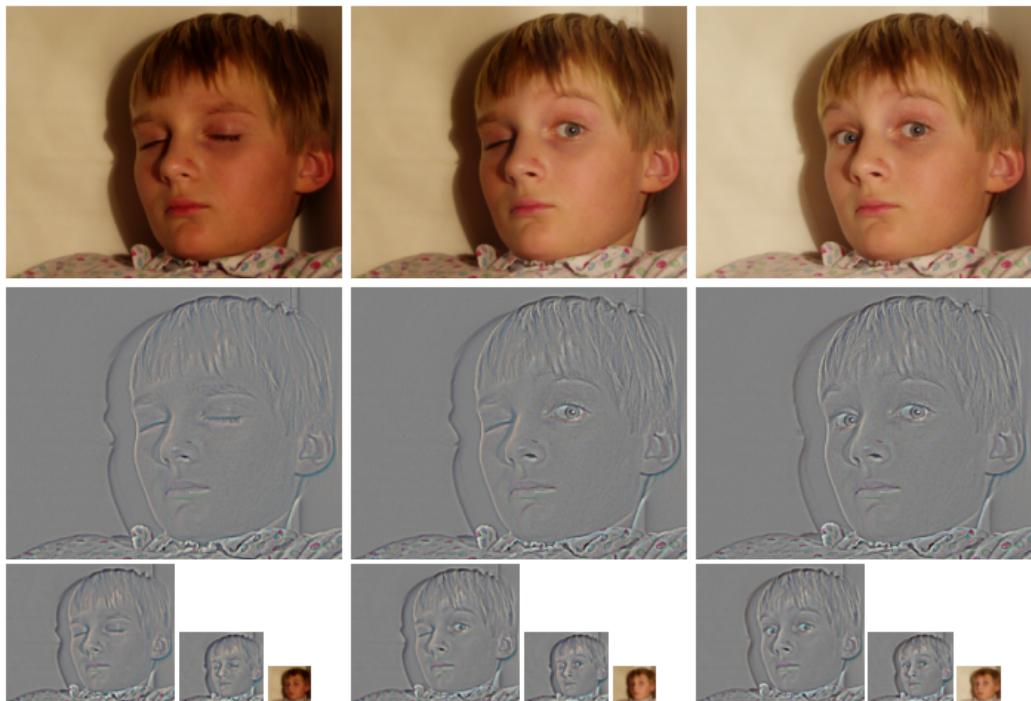
Band-pass image analysis and synthesis using Laplacian pyramid:



Difference of Gaussians \approx Laplacian of Gaussian (band-pass filter):

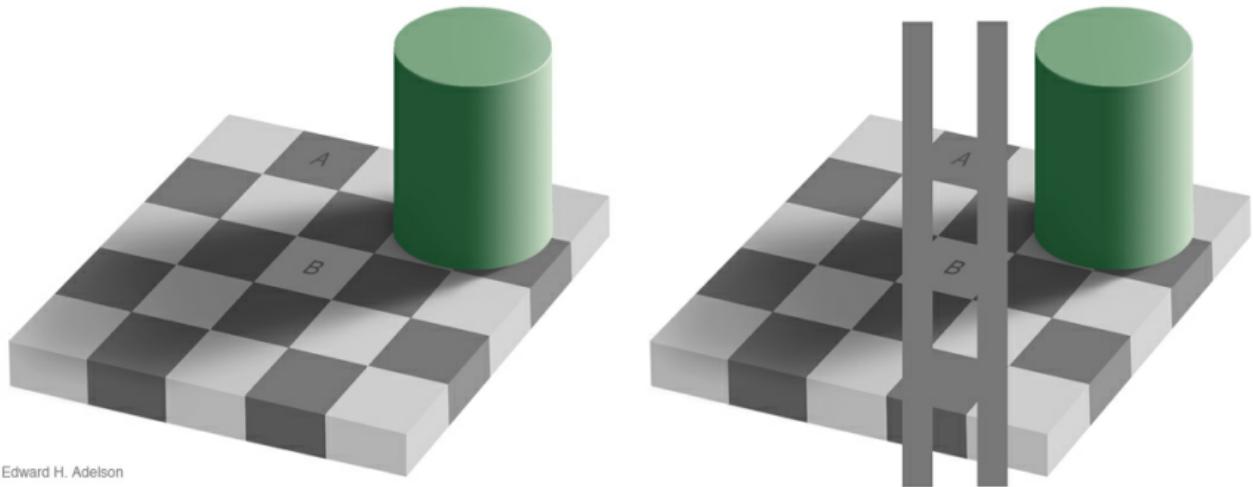


Blend each level of Laplacian pyramid and synthesize the image:



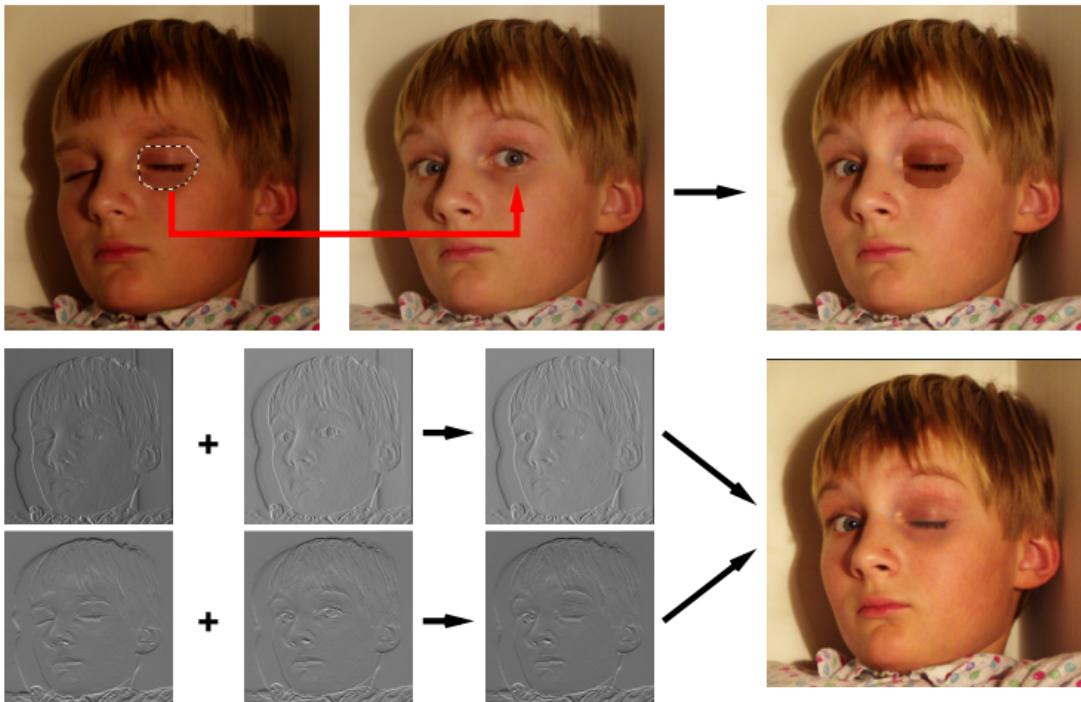
Human visual system is not very sensitive to absolute intensity.

Adelson's checkerboard illusion:



Local intensity changes (gradients) are much more important.

New approach: modify gradients and then reconstruct intensity.



Problem: only conservative gradient field can be integrated in 2D.

$$\nabla I = G \quad \Rightarrow \quad I = \iint G \quad \Leftrightarrow \quad \oint G = 0$$

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Solution: find an image I^* of which gradient field is close to G .

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$$\nabla \textcolor{red}{I} = \textcolor{blue}{G} \quad \Rightarrow \quad \textcolor{red}{I} = \iint \textcolor{blue}{G} \quad \Leftrightarrow \quad \oint \textcolor{blue}{G} = 0$$

Solution: find an image $\textcolor{red}{I}^*$ of which gradient field is close to $\textcolor{blue}{G}$.

$$\textcolor{red}{I}^* = \arg \min_{\textcolor{red}{I}} \iint ||\nabla \textcolor{red}{I} - \textcolor{blue}{G}||^2$$

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$$\textcolor{red}{I}^* = \arg \min_{\textcolor{red}{I}} \iint \left(\frac{\partial \textcolor{red}{I}}{\partial x} - \textcolor{blue}{G}_x \right)^2 + \left(\frac{\partial \textcolor{red}{I}}{\partial y} - \textcolor{blue}{G}_y \right)^2$$

We want to minimize:

$$\iint \underbrace{\left(\frac{\partial \textcolor{red}{I}}{\partial x} - \textcolor{blue}{G}_x \right)^2 + \left(\frac{\partial \textcolor{red}{I}}{\partial y} - \textcolor{blue}{G}_y \right)^2}_{\textcolor{green}{F}}$$

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Variational Principle (Euler-Lagrange equation):

$$\frac{\partial \textcolor{green}{F}}{\partial \textcolor{red}{I}} - \frac{d}{dx} \frac{\partial \textcolor{green}{F}}{\partial \textcolor{red}{I}_x} - \frac{d}{dy} \frac{\partial \textcolor{green}{F}}{\partial \textcolor{red}{I}_y} = 0 \quad \Rightarrow \quad 2 \left(\frac{\partial^2 \textcolor{red}{I}}{\partial x^2} - \frac{\partial \textcolor{blue}{G}_x}{\partial x} \right) + 2 \left(\frac{\partial^2 \textcolor{red}{I}}{\partial y^2} - \frac{\partial \textcolor{blue}{G}_y}{\partial y} \right) = 0$$

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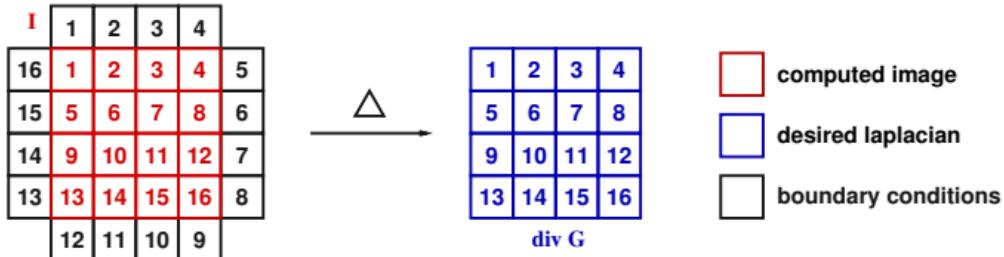
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Poisson equation:

$$\frac{\partial^2 \textcolor{red}{I}}{\partial x^2} + \frac{\partial^2 \textcolor{red}{I}}{\partial y^2} = \Delta \textcolor{red}{I} \quad \boxed{\Delta \textcolor{red}{I} = \operatorname{div} \textcolor{blue}{G}} \quad \operatorname{div} \textcolor{blue}{G} = \frac{\partial \textcolor{blue}{G}_x}{\partial x} + \frac{\partial \textcolor{blue}{G}_y}{\partial y}$$

Discretization of Poisson equation:



$$\begin{array}{ccccccccc}
 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 \\
 \hline
 -4 & 1 & & & 1 & & & & & & & & & & & \\
 1 & -4 & 1 & & & 1 & & & & & & & & & & \\
 & 1 & -4 & 1 & & & 1 & & & & & & & & & \\
 & & 1 & -4 & & & & 1 & & & & & & & & \\
 1 & & & -4 & 1 & & & 1 & & & & & & & & \\
 1 & & & 1 & -4 & 1 & & & 1 & & & & & & & \\
 & 1 & & 1 & 1 & -4 & 1 & & & 1 & & & & & & \\
 & & 1 & & 1 & 1 & -4 & & & & 1 & & & & &
 \end{array} \cdot \begin{array}{l}
 1 = 1 - 1 - 16 \\
 2 = 2 - 2 - 5 \\
 3 = 3 - 3 - 3 \\
 4 = 4 - 4 - 15 \\
 5 = 5 - 6 - 15 \\
 6 = 6 - 7 - 6 \\
 7 = 7 - 8 - 6 \\
 8 = 8 - 6 - 6
 \end{array}$$

Large but very sparse system of linear equations:

$$\mathbf{A}_{[N^2 \times N^2]} \cdot \mathbf{I}_{[N^2]} = \mathbf{b}_{[N^2]}$$

A not stored in memory \Rightarrow direct computation

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Gauss-Seidel:

$$\mathbf{I}'[x, y] = \frac{1}{4} (\mathbf{I}[x+1, y] + \mathbf{I}[x-1, y] + \mathbf{I}[x, y+1] + \mathbf{I}[x, y-1] - \mathbf{b}[x, y])$$

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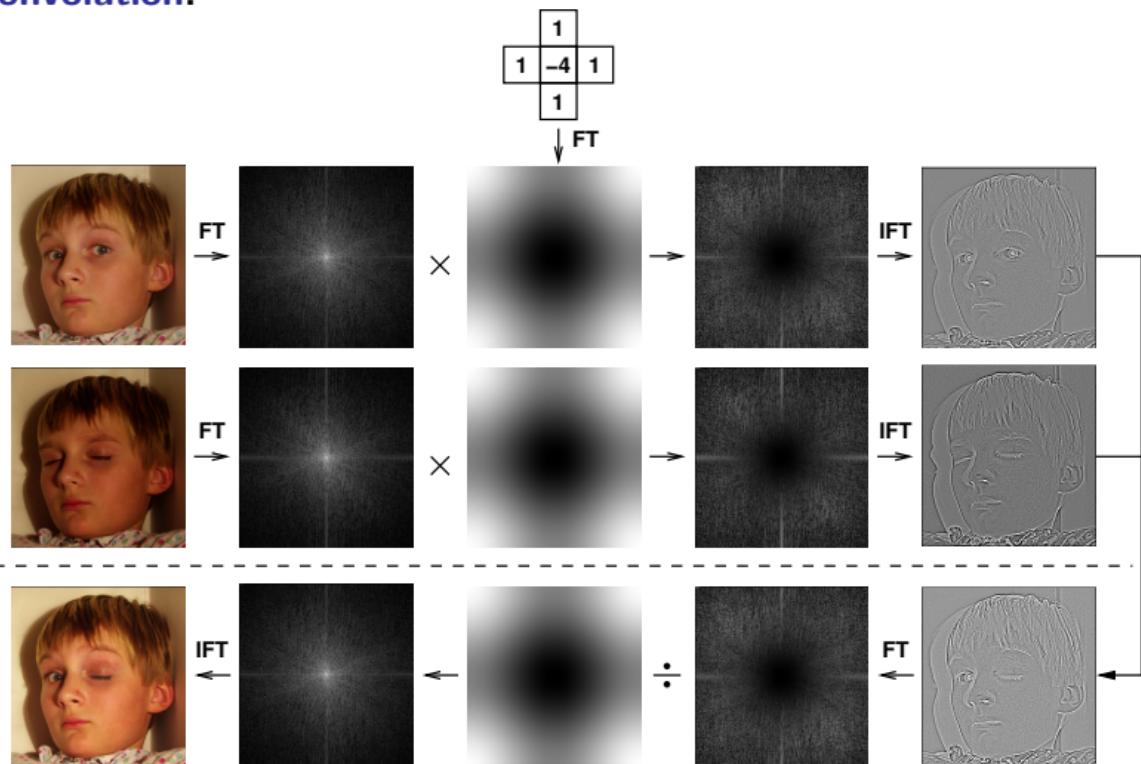
Conjugate gradient:

$$\mathbf{I}' = \mathbf{I} + \alpha \cdot \mathbf{g} \quad \mathbf{g} = \mathbf{b} - \mathbf{A} \cdot \mathbf{I} \quad \alpha = \frac{\|\mathbf{g}\|}{\mathbf{g}^T \mathbf{A} \mathbf{g}}$$

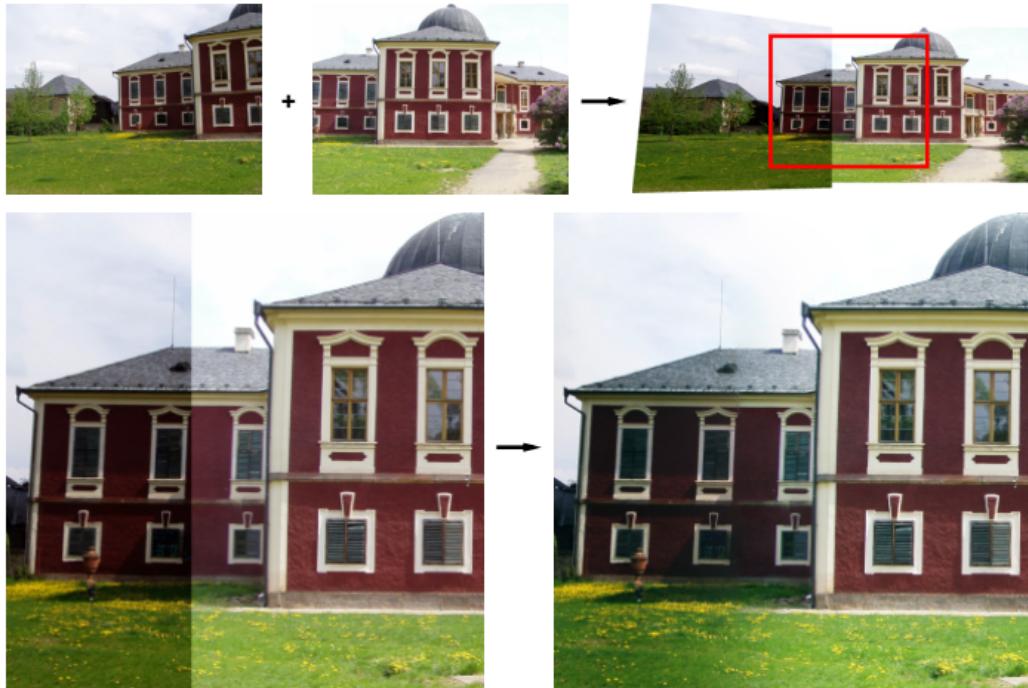
Multi-resolution scheme:



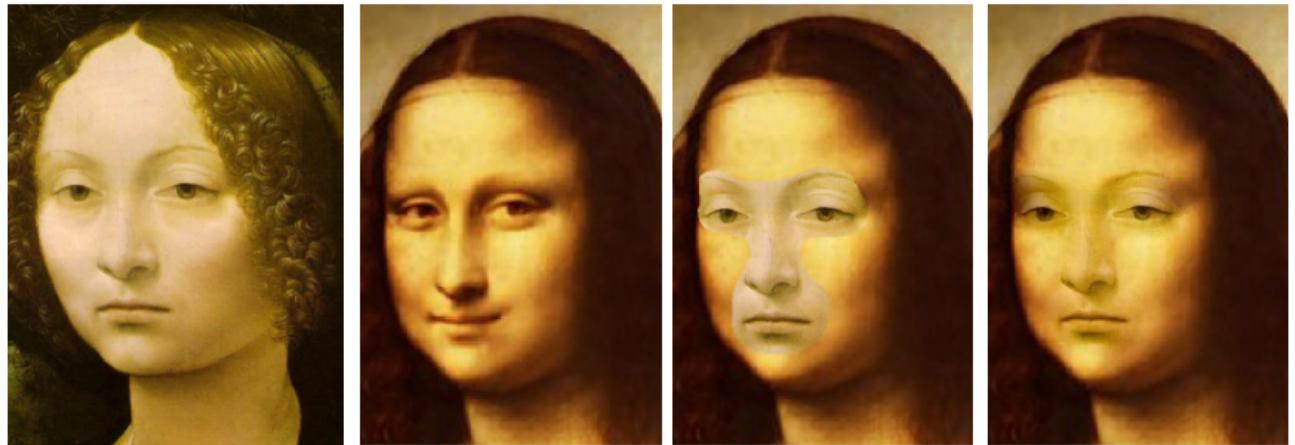
Compute initial solution at lower resolution and refine.

Deconvolution:

seamless panorama:



seamless cloning:



seamless concealment:



HDR image synthesis:



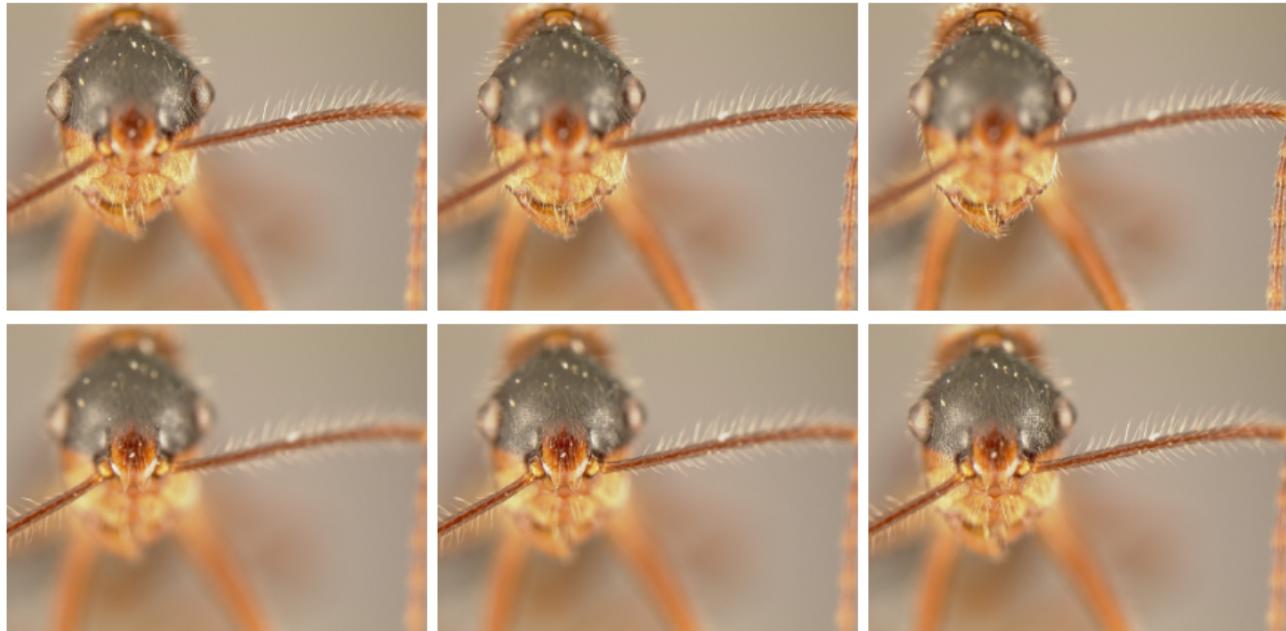
+



HDR image synthesis:



sharp image synthesis:



sharp image synthesis:



group photo synthesis:



group photo synthesis:



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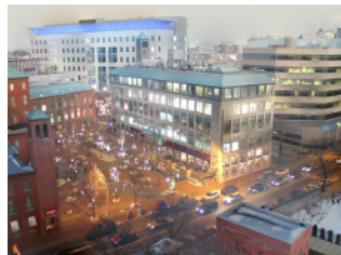
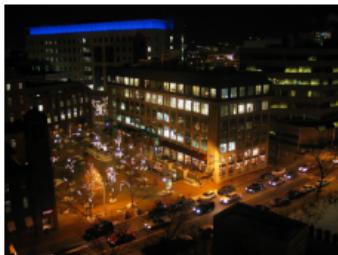
group photo synthesis:



group photo synthesis:



context enhancement and surrealism:



color to gray-scale conversion:



intensity



gradients



diffusion curves:

