Digital Image

(B4M33DZO, Winter 2024)

Lecture 8:

Image Registration 1

https://cw.fel.cvut.cz/wiki/courses/b4m33dzo/start

Daniel Sýkora & Ondřej Drbohlav

Department of Cybernetics

Faculty of Electrical Engineering

Czech Technical University in Prague

Image similarity

Sum of absolute differences (SAD): $$\sum_x \sum_y |\mathbf{A}[x,y] - \mathbf{B}[x,y]|$$

Image similarity

Sum of absolute differences (SAD): $$\sum_x \sum_y |\mathbf{A}[x,y] - \mathbf{B}[x,y]|$$

Sum of squared differences (SSD): $$\sum_x \sum_y (\mathbf{A}[x,y] - \mathbf{B}[x,y])^2$$

Image similarity

Sum of absolute differences (SAD): $$\sum_x \sum_y |\mathbf{A}[x,y] - \mathbf{B}[x,y]|$$

Sum of squared differences (SSD): $$\sum_x \sum_y (\mathbf{A}[x,y] - \mathbf{B}[x,y])^2$$

$$\sum_x \sum_y \mathbf{A}[x,y]^2 - 2 \cdot \mathbf{A}[x,y] \cdot \mathbf{B}[x,y] + \mathbf{B}[x,y]^2$$

Image similarity

Sum of absolute differences (SAD): $\sum_x \sum_y |\mathbf{A}[x,y] - \mathbf{B}[x,y]|$

Sum of squared differences (SSD): $\sum_x \sum_y (\mathbf{A}[x,y] - \mathbf{B}[x,y])^2$

$$\sum_x \sum_y \mathbf{A}[x,y]^2 - 2 \cdot \underbrace{\mathbf{A}[x,y] \cdot \mathbf{B}[x,y]} + \mathbf{B}[x,y]^2$$

Cross-correlation:

$$\sum_x \sum_y \mathbf{A}(x,y) \cdot \mathbf{B}(x,y)$$

Image similarity

Sum of absolute differences (SAD): $$\sum_x \sum_y |\mathbf{A}[x,y] - \mathbf{B}[x,y]|$$

Sum of squared differences (SSD): $$\sum_x \sum_y (\mathbf{A}[x,y] - \mathbf{B}[x,y])^2$$

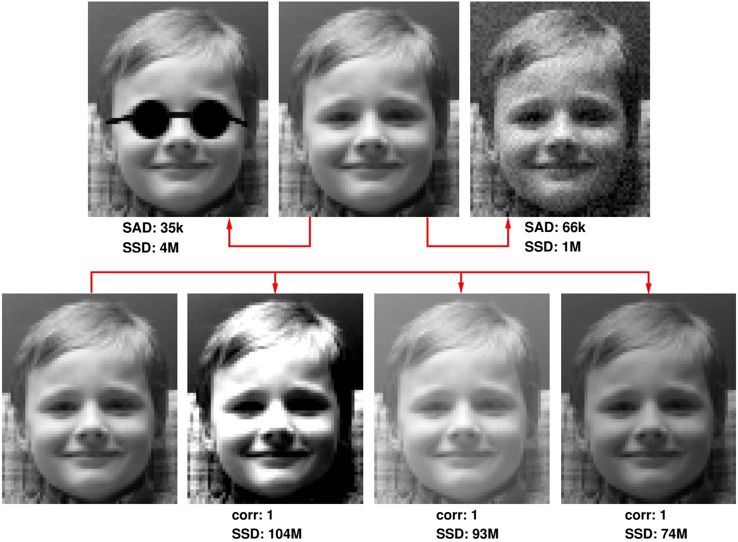
$$\sum_x \sum_y \mathbf{A}[x,y]^2 - 2 \cdot \mathbf{A}[x,y] \cdot \mathbf{B}[x,y] + \mathbf{B}[x,y]^2$$

Normalized cross-correlation:

$$\frac{1}{\sigma_{\mathbf{A}} \sigma_{\mathbf{B}}} \sum_x \sum_y \left(\mathbf{A}(x,y) - \hat{\mathbf{A}}\right) \cdot \left(\mathbf{B}(x,y) - \hat{\mathbf{B}}\right)$$

(invariant to brightness & contrast changes)

Image similarity



Translation

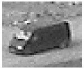
Minimize similarity metric ∘ over all possible shifts:

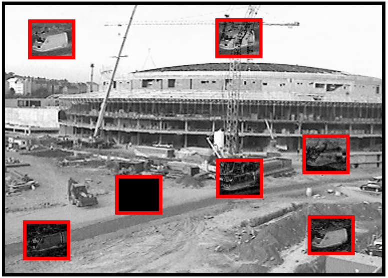
$$\arg\min_{[s,t]} \sum_x \sum_y \mathbf{A}[x+s, y+t] \circ \mathbf{B}[x,y]$$

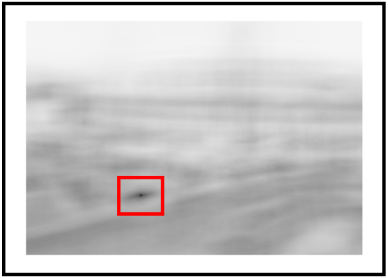
Translation

Minimize similarity metric $\circ$ over all possible shifts:

$$\arg\min_{[s,t]} \sum_x \sum_y \mathbf{A}[x+s, y+t] \circ \mathbf{B}[x,y]$$



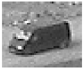


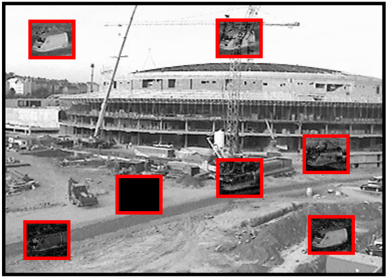


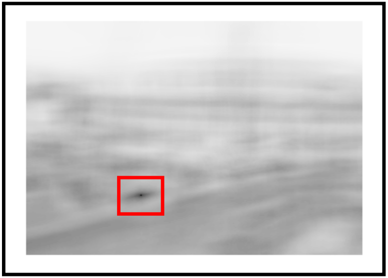
Translation

Minimize similarity metric $\circ$ over all possible shifts:

$$\arg\min_{[s,t]} \sum_x \sum_y \mathbf{A}[x+s, y+t] \circ \mathbf{B}[x,y]$$







Problem: complexity of block matching is $\mathcal{O}(|\mathbf{A}| \cdot |\mathbf{B}|)$.

Translation (speed-ups)

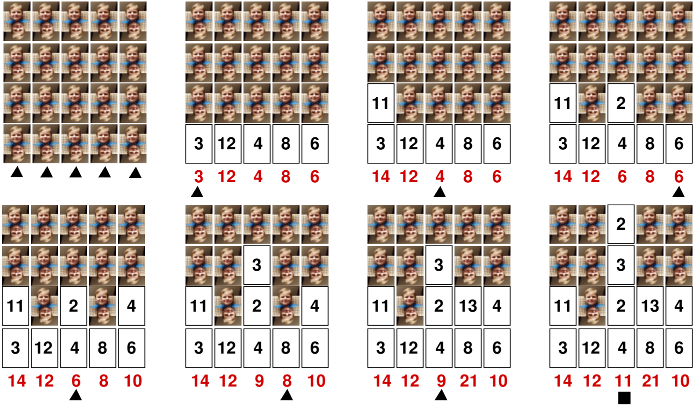
1. **Early termination** ($\mathcal{O}(|\mathbf{A}| \cdot |\mathbf{B}|)$, global minimum):
   Compare current summation with the last best value.

Translation (speed-ups)

1. **Early termination** ($\mathcal{O}(|\mathbf{A}| \cdot |\mathbf{B}|)$, **global minimum):**

   Compare current summation with the last best value.

2. **Winner-update strategy** ($\mathcal{O}(|\mathbf{A}| \cdot |\mathbf{B}|)$, **global minimum):**



Translation (speed-ups)

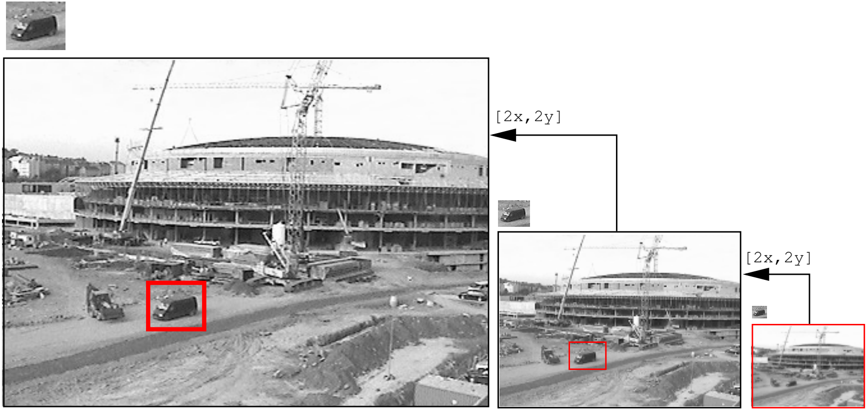
3. Hierarchical approach ($\mathcal{O}(|\mathbf{A}| \cdot \log |\mathbf{A}|)$, local minimum):

Use reduced resolution to compute initial solution and refine.

Translation (speed-ups)

3. **Hierarchical approach** ($\mathcal{O}(|\mathbf{A}| \cdot \log |\mathbf{A}|)$, **local minimum):**

Use reduced resolution to compute initial solution and refine.



Translation (speed-ups)

4. Phase correlation ($\mathcal{O}(|\mathbf{A}| \cdot \log |\mathbf{A}|)$, local minimum):

$$\mathbf{A}[x, y] * \delta(s, t) = \mathbf{B}[x, y] \quad \Longleftrightarrow \quad \mathcal{A}[u, v] \cdot e^{2\pi i(us+vt)} = \mathcal{B}[u, v]$$

4. Phase correlation ($\mathcal{O}(|\mathbf{A}| \cdot \log |\mathbf{A}|)$, local minimum):

$$\mathbf{A}[x,y] * \delta(s,t) = \mathbf{B}[x,y] \quad \Longleftrightarrow \quad \mathcal{A}[u,v] \cdot e^{2\pi i(us+vt)} = \mathcal{B}[u,v]$$

$$\frac{\mathcal{A}^*[u,v] \cdot \mathcal{B}[u,v]}{|\mathcal{A}[u,v]|^2} = e^{2\pi i(us+vt)} \quad \Longrightarrow \quad \delta(s,t)$$

Translation (speed-ups)

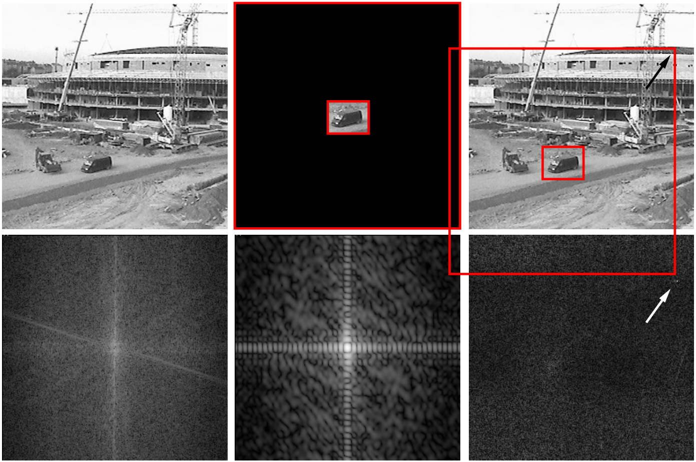
4. Phase correlation ($\mathcal{O}(|\mathbf{A}| \cdot \log |\mathbf{A}|)$, local minimum):

$$\mathbf{A}[x,y] * \delta(s,t) = \mathbf{B}[x,y] \quad \Longleftrightarrow \quad \mathcal{A}[u,v] \cdot e^{2\pi i(us+vt)} = \mathcal{B}[u,v]$$

$$\frac{\mathcal{A}^*[u,v] \cdot \mathcal{B}[u,v]}{|\mathcal{A}[u,v] \cdot \mathcal{B}[u,v]|} = e^{2\pi i(us+vt)} \quad \Longrightarrow \quad \delta(s,t)$$

Translation (speed-ups)

4. Phase correlation ($\mathcal{O}(|\mathbf{A}| \cdot \log |\mathbf{A}|)$, local minimum):



Translation (speed-ups)

5. SSD decomposition ($\mathcal{O}(|\mathbf{A}| \cdot \log |\mathbf{A}|)$, global minimum):

$$\arg\min_{[s,t]} \sum_{x=0}^{w} \sum_{y=0}^{h} (\mathbf{A}[x+s, y+t] - \mathbf{B}[x, y])^2 =$$

Translation (speed-ups)

5. SSD decomposition ($\mathcal{O}(|\mathbf{A}| \cdot \log |\mathbf{A}|)$, global minimum):

$$\arg\min_{[s,t]} \sum_{x=0}^{w} \sum_{y=0}^{h} (\mathbf{A}[x+s, y+t] - \mathbf{B}[x,y])^2 =$$

$$\arg\min_{[s,t]} \left(\sum_{x=s}^{s+w} \sum_{y=t}^{t+h} \mathbf{A}[x,y]^2 - 2 \sum_{x=0}^{w} \sum_{y=0}^{h} \mathbf{A}[x+s, y+t] \cdot \mathbf{B}[x,y] \right)$$

Translation (speed-ups)

5. SSD decomposition ($\mathcal{O}(|\mathbf{A}| \cdot \log |\mathbf{A}|)$, global minimum):

$$\arg\min_{[s,t]} \sum_{x=0}^{w} \sum_{y=0}^{h} \left(\mathbf{A}[x+s, y+t] - \mathbf{B}[x,y]\right)^2 =$$

$$\arg\min_{[s,t]} \left(\sum_{x=s}^{s+w} \sum_{y=t}^{t+h} \mathbf{A}[x,y]^2 - 2 \sum_{x=0}^{w} \sum_{y=0}^{h} \mathbf{A}[x+s, y+t] \cdot \mathbf{B}[x,y] \right)$$

Summed area table:

$$\sum_{x=s}^{s+w} \sum_{y=t}^{t+h} \mathbf{A}[x,y]^2 = \Sigma[s,t] - \Sigma[s+w,t] - \Sigma[s,t+h] + \Sigma[s+w,t+h]$$

Translation (speed-ups)

5. SSD decomposition ($\mathcal{O}(|\mathbf{A}| \cdot \log |\mathbf{A}|)$, global minimum):

$$\arg\min_{[s,t]} \sum_{x=0}^{w} \sum_{y=0}^{h} \left(\mathbf{A}[x+s, y+t] - \mathbf{B}[x,y]\right)^2 =$$

$$\arg\min_{[s,t]} \left(\sum_{x=s}^{s+w} \sum_{y=t}^{t+h} \mathbf{A}[x,y]^2 - 2 \sum_{x=0}^{w} \sum_{y=0}^{h} \mathbf{A}[x+s, y+t] \cdot \mathbf{B}[x,y] \right)$$

Summed area table:

$$\sum_{x=s}^{s+w} \sum_{y=t}^{t+h} \mathbf{A}[x,y]^2 = \Sigma[s,t] - \Sigma[s+w,t] - \Sigma[s,t+h] + \Sigma[s+w,t+h]$$

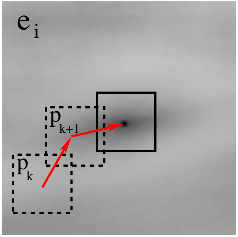
Fourier convolution theorem:

$$\sum_{x=0}^{w} \sum_{y=0}^{h} \mathbf{A}[x+s, y+t] \cdot \mathbf{B}[x,y] = \mathcal{F}^{-1}\left\{\mathcal{F}\{\mathbf{A}\} \cdot \mathcal{F}\{\mathbf{B}\}\right\}[s,t]$$

6. Gradient descent ($\mathcal{O}(|\mathrm{B}|)$, local minimum):

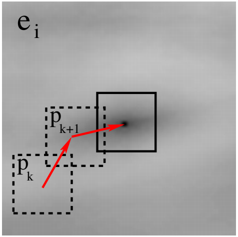
We want to minimize:

$$E = \sum_i \left(\mathbf{A}[\mathbf{x}_i + \mathbf{t}] - \mathbf{B}[\mathbf{x}_i]\right)^2 \quad \Rightarrow \quad E' = 0$$



Translation (speed-ups)

6. Gradient descent ($\mathcal{O}(|\mathbf{B}|)$, local minimum):



We want to minimize:

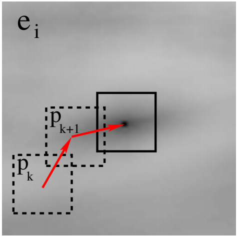
$$E = \sum_i \left(\mathbf{A}[\mathbf{x}_i + \mathbf{t}] - \mathbf{B}[\mathbf{x}_i]\right)^2 \quad \Rightarrow \quad E' = 0$$

Using linear approximation:

$$\mathbf{A}[\mathbf{x}_i + \mathbf{t}] \approx \mathbf{A}[\mathbf{x}_i] + \frac{\partial}{\partial \mathbf{x}}\mathbf{A}[\mathbf{x}_i] \cdot \mathbf{t}$$

Translation (speed-ups)

6. Gradient descent ($\mathcal{O}(|\mathbf{B}|)$, local minimum):



We want to minimize:

$$E = \sum_i \left(\mathbf{A}[\mathbf{x}_i + \mathbf{t}] - \mathbf{B}[\mathbf{x}_i]\right)^2 \quad \Rightarrow \quad E' = 0$$

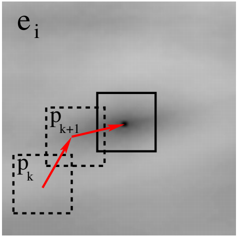
Using linear approximation:

$$\mathbf{A}[\mathbf{x}_i + \mathbf{t}] \approx \mathbf{A}[\mathbf{x}_i] + \frac{\partial}{\partial \mathbf{x}}\mathbf{A}[\mathbf{x}_i] \cdot \mathbf{t}$$

$$\mathbf{A}[\mathbf{x}_i] \rightarrow \mathbf{A}_i \quad \mathbf{B}[\mathbf{x}_i] \rightarrow \mathbf{B}_i \quad \frac{\partial}{\partial \mathbf{x}}\mathbf{A}_i \rightarrow \mathbf{A}'_i \quad \frac{\partial E}{\partial \mathbf{t}} \rightarrow E'$$

Translation (speed-ups)

6. Gradient descent ($\mathcal{O}(|\mathbf{B}|)$, local minimum):



We want to minimize:

$$E = \sum_i \left(\mathbf{A}[\mathbf{x}_i + \mathbf{t}] - \mathbf{B}[\mathbf{x}_i]\right)^2 \quad \Rightarrow \quad E' = 0$$

Using linear approximation:

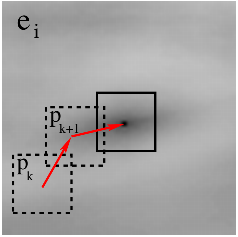
$$\mathbf{A}[\mathbf{x}_i + \mathbf{t}] \approx \mathbf{A}[\mathbf{x}_i] + \frac{\partial}{\partial \mathbf{x}}\mathbf{A}[\mathbf{x}_i] \cdot \mathbf{t}$$

$$\mathbf{A}[\mathbf{x}_i] \to \mathbf{A}_i \quad \mathbf{B}[\mathbf{x}_i] \to \mathbf{B}_i \quad \frac{\partial}{\partial \mathbf{x}}\mathbf{A}_i \to \mathbf{A}'_i \quad \frac{\partial E}{\partial \mathbf{t}} \to E'$$

$$E' \approx \frac{\partial}{\partial \mathbf{t}} \sum_i \left(\mathbf{A}_i + \mathbf{A}'_i\mathbf{t} - \mathbf{B}_i\right)^2 = 2\sum_i \left(\mathbf{A}'_i\right)^T \left(\mathbf{A}_i + \mathbf{A}'_i\mathbf{t} - \mathbf{B}_i\right) = 0$$

Translation (speed-ups)

6. Gradient descent ($\mathcal{O}(|\mathbf{B}|)$, local minimum):



We want to minimize:

$$E = \sum_i \left(\mathbf{A}[\mathbf{x}_i + \mathbf{t}] - \mathbf{B}[\mathbf{x}_i]\right)^2 \quad \Rightarrow \quad E' = 0$$

Using linear approximation:

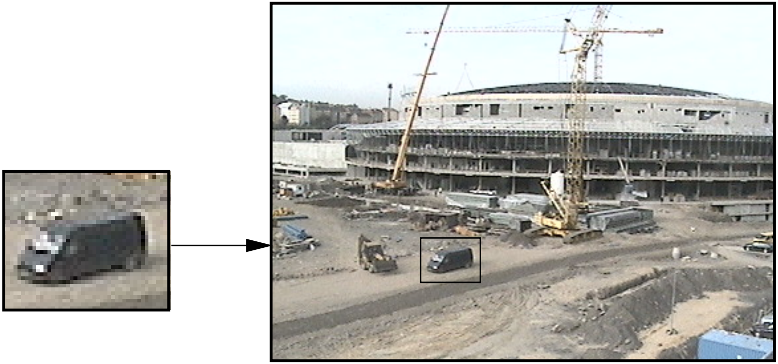
$$\mathbf{A}[\mathbf{x}_i + \mathbf{t}] \approx \mathbf{A}[\mathbf{x}_i] + \frac{\partial}{\partial \mathbf{x}}\mathbf{A}[\mathbf{x}_i] \cdot \mathbf{t}$$

$$\mathbf{A}[\mathbf{x}_i] \rightarrow \mathbf{A}_i \quad \mathbf{B}[\mathbf{x}_i] \rightarrow \mathbf{B}_i \quad \frac{\partial}{\partial \mathbf{x}}\mathbf{A}_i \rightarrow \mathbf{A}'_i \quad \frac{\partial E}{\partial \mathbf{t}} \rightarrow E'$$

$$E' \approx \frac{\partial}{\partial \mathbf{t}} \sum_i \left(\mathbf{A}_i + \mathbf{A}'_i\mathbf{t} - \mathbf{B}_i\right)^2 = 2\sum_i \left(\mathbf{A}'_i\right)^T \left(\mathbf{A}_i + \mathbf{A}'_i\mathbf{t} - \mathbf{B}_i\right) = 0$$

$$\mathbf{t} = \left(\sum_i \left(\mathbf{A}'_i\right)^T \left(\mathbf{A}'_i\right)\right)^{-1} \left(\sum_i \left(\mathbf{A}'_i\right)^T \left(\mathbf{B}_i - \mathbf{A}_i\right)\right)$$

Applications



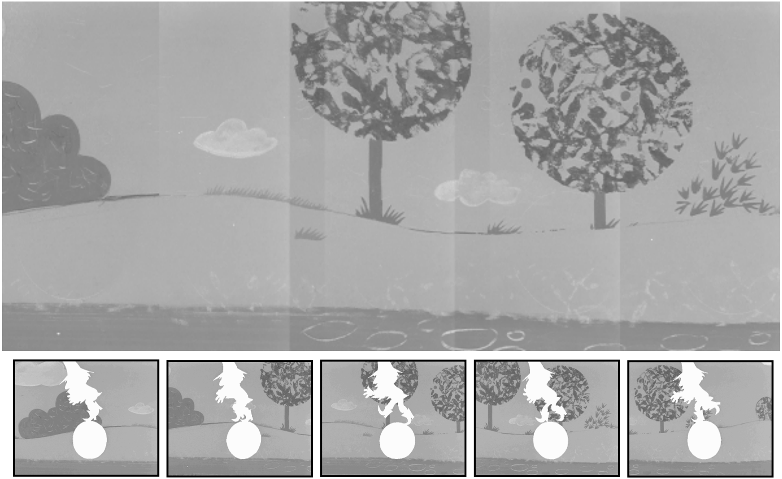
template matching, motion estimation, video compression, . . .

Applications

stitching, stabilization, restoration, retrieval, ...

Applications

Recovering background from occluded observations:



Applications

hole filling & texture synthesis