Digital Image

(B4M33DZO, Winter 2024)

Lecture 9:

Image Registration 2

https://cw.fel.cvut.cz/wiki/courses/b4m33dzo/start

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Rotation

Polar transformation:

rotation $\Longrightarrow$ **translation on y-axis**

Rotation

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rotation $\Longrightarrow$ translation on y-axis

$$r = \sqrt{(x - x_c)^2 + (y - y_c)^2}$$

$$\alpha = \arctan \frac{y - y_c}{x - x_c}$$

Rotation

Polar transformation:

rotation $\implies$ translation on y-axis

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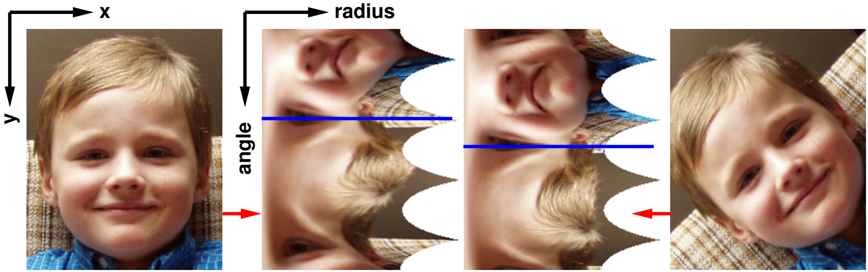


Rotation & Scale

Log-polar transformation:

scale $\Longrightarrow$ **translation on x-axis**
rotation $\Longrightarrow$ **translation on y-axis**

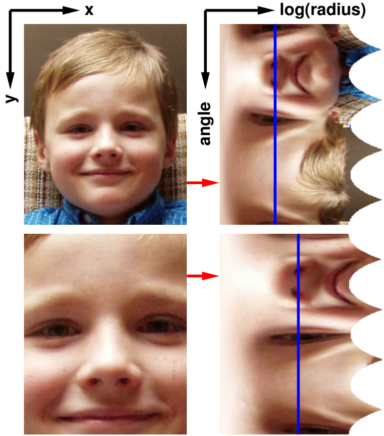
Rotation & Scale

Log-polar transformation:

scale $\Longrightarrow$ **translation on x-axis**
rotation $\Longrightarrow$ **translation on y-axis**

$$r = \log\left(1 + \sqrt{(x - x_c)^2 + (y - y_c)^2}\right)$$

$$\alpha = \arctan \frac{y - y_c}{x - x_c}$$



Fourier-Mellin transformation:

1. **Estimate rotation and scale using frequency domain.**
2. **Resample image to have the same rotation and scale.**
3. **Use standard phase correlation to estimate translation.**

Rotation & Scale & Translation

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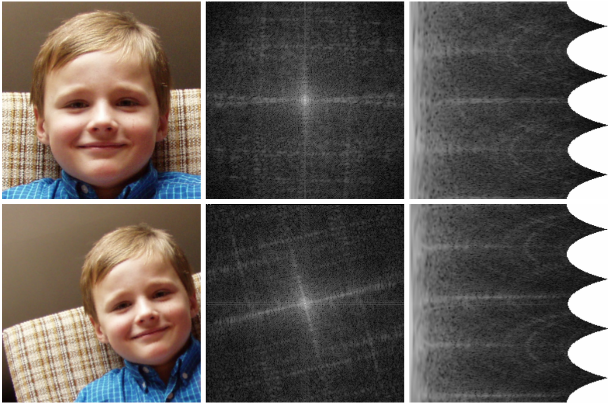

Affine & Projective

Generalized gradient descent:

$$E = \sum_i \left(\mathbf{A}[\mathbf{W}(\mathbf{x}_i, \mathbf{p})] - \mathbf{B}[\mathbf{x}_i]\right)^2 \approx \sum_i \left(\mathbf{A}_i + \mathbf{A}'_i \mathbf{J} \mathbf{p} - \mathbf{B}_i\right)^2$$

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$$\mathbf{A}_i \to \mathbf{A}[\mathbf{x}_i] \quad \mathbf{B}_i \to \mathbf{B}[\mathbf{x}_i] \quad \mathbf{A}'_i \to \frac{\partial}{\partial \mathbf{x}} \mathbf{A}_i$$

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$$E' = 2 \sum_i \left(\mathbf{A}'_i \mathbf{J}\right)^T \left(\mathbf{A}_i + \mathbf{A}'_i \mathbf{J} \mathbf{p} - \mathbf{B}_i\right) = 0$$

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$$\mathbf{p} = \mathbf{H}^{-1} \sum_i \left(\mathbf{A}'_i \mathbf{J}\right)^T \left(\mathbf{B}_i - \mathbf{A}_i\right)$$

Affine & Projective

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$$\mathbf{p} = \mathbf{H}^{-1} \sum_i \left(\mathbf{A}'_i \mathbf{J}\right)^T (\mathbf{B}_i - \mathbf{A}_i)$$

$$\mathbf{J} = \underbrace{\begin{pmatrix} \frac{\partial \mathbf{W}_\mathbf{x}}{\partial \mathbf{p}_1} & \frac{\partial \mathbf{W}_\mathbf{x}}{\partial \mathbf{p}_2} & \cdots & \frac{\partial \mathbf{W}_\mathbf{x}}{\partial \mathbf{p}_n} \\ \frac{\partial \mathbf{W}_\mathbf{y}}{\partial \mathbf{p}_1} & \frac{\partial \mathbf{W}_\mathbf{y}}{\partial \mathbf{p}_2} & \cdots & \frac{\partial \mathbf{W}_\mathbf{y}}{\partial \mathbf{p}_n} \end{pmatrix}}_{\text{Jacobian}} \qquad \mathbf{H} = \underbrace{\sum_i \left(\mathbf{A}'_i \mathbf{J}\right)^T \left(\mathbf{A}'_i \mathbf{J}\right)}_{\text{Hessian}}$$

Jacobian

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Affine transformation: $\mathbf{p} = (a_{11}, a_{12}, a_{21}, a_{22}, x_0, y_0)$

$$x' = a_{11}x + a_{12}y + x_0 \qquad y' = a_{21}x + a_{22}y + y_0$$

Affine & Projective

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Projective transformation: $\mathbf{p} = (a_{11}, a_{12}, a_{21}, a_{22}, x_0, y_0, f_1, f_2)$

$$x' = \frac{a_{11}x + a_{12}y + x_0}{f_1x + f_2y + 1} \qquad y' = \frac{a_{21}x + a_{22}y + y_0}{f_1x + f_2y + 1}$$

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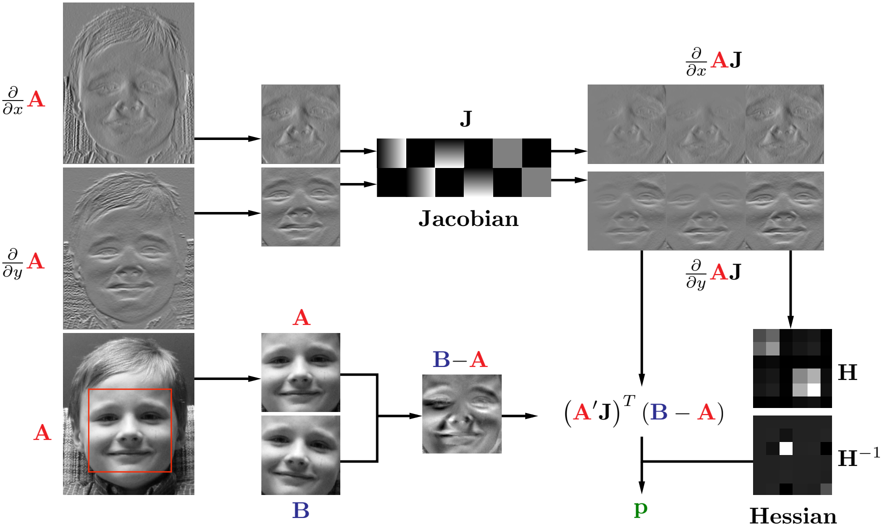
$$\mathbf{J} = \frac{1}{f_1 x + f_2 y + 1} \begin{pmatrix} x & 0 & -xx' & y & 0 & -yx' & 1 & 0 \\ 0 & x & -xy' & 0 & y & -yy' & 0 & 1 \end{pmatrix}$$

Affine



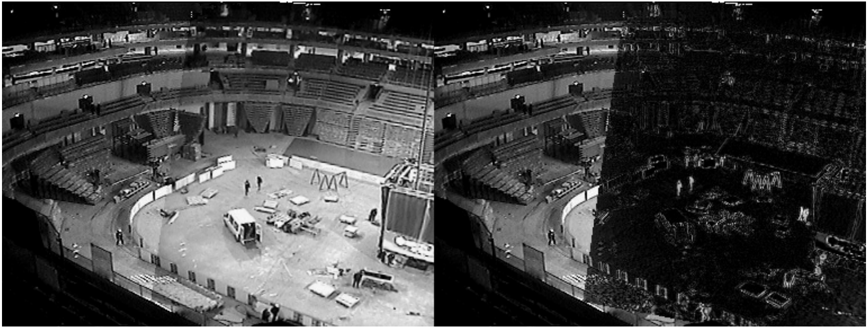
$\frac{\partial}{\partial x}\mathbf{A}$
$\frac{\partial}{\partial y}\mathbf{A}$
$\mathbf{A}$
$\mathbf{A}$
$\mathbf{B}$
$\mathbf{J}$
Jacobian
$\mathbf{B}-\mathbf{A}$
$\frac{\partial}{\partial x}\mathbf{A}\mathbf{J}$
$\frac{\partial}{\partial y}\mathbf{A}\mathbf{J}$
$\left(\mathbf{A}'\mathbf{J}\right)^T(\mathbf{B}-\mathbf{A})$
$\mathbf{H}$
$\mathbf{H}^{-1}$
$\mathbf{p}$
Hessian

Hierarchical approach

Hierarchical gradient descent (Lucas-Kanade):

1. **Model: translation ⇒ rigid ⇒ affine ⇒ projective**
2. **Scale: 32x32 ⇒ 64x64 ⇒ 128x128 ⇒ 256x256**

Composite **Error**



Deformable Registration

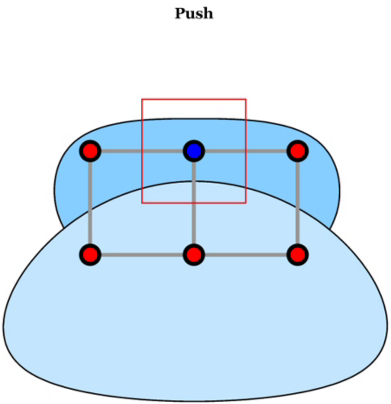
Algorithm:

repeat

1. **Push:**
   block matching

2. Regularize:
   shape matching

until convergence



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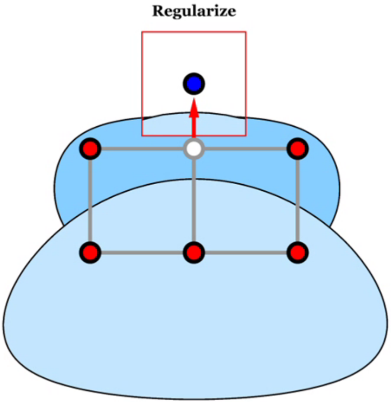

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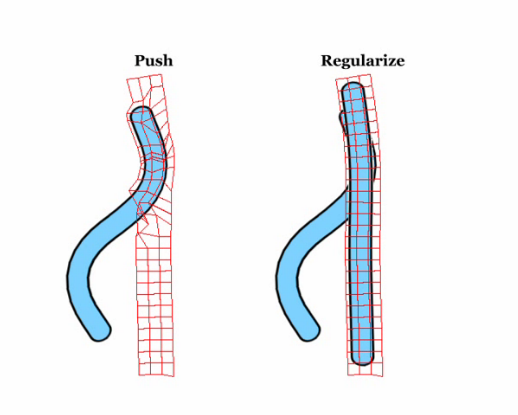

Deformable Registration

Applications

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