

# Digital Image

(B4M33DZO, Summer 2024)

## Lecture 10:

### Image Registration 3

<https://cw.fel.cvut.cz/wiki/courses/b4m33dzo/start>

**Daniel Sýkora & Ondřej Drbohlav**

Department of Cybernetics

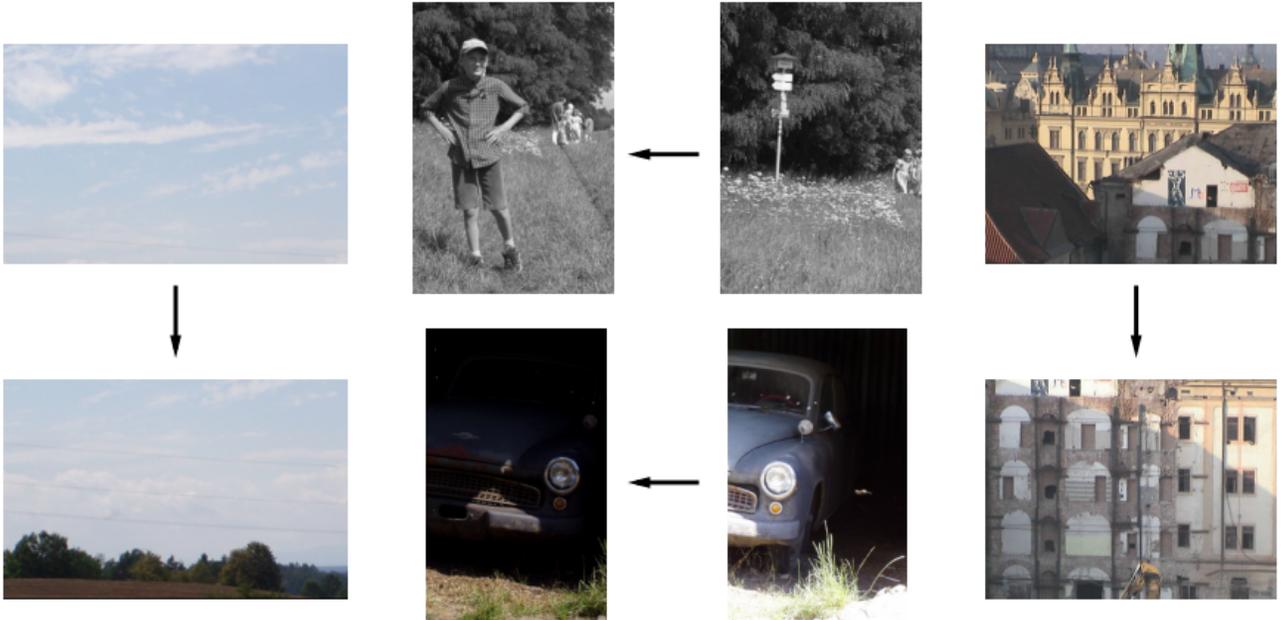
Faculty of Electrical Engineering

Czech Technical University in Prague

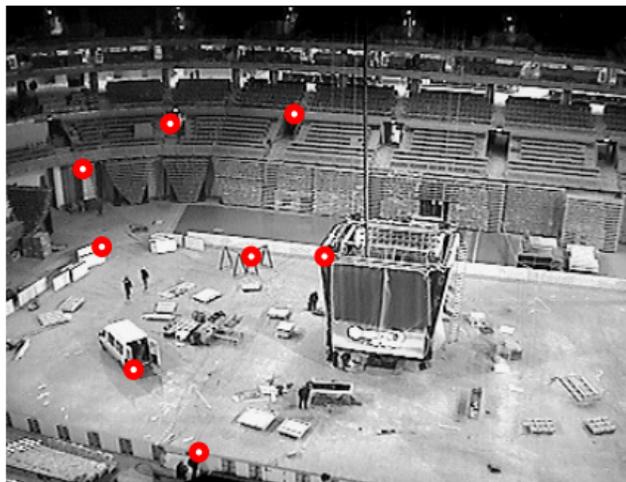
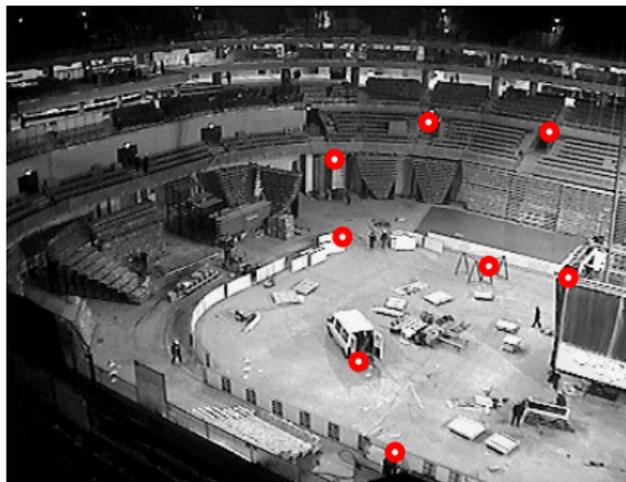
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## Real world and image registration:



**lack of features, occlusion, different exposure, small overlap**



In contrast to image-based approach:

**Efficiency:** significant problem reduction (e.g. 1k point vs. 1M pixel)

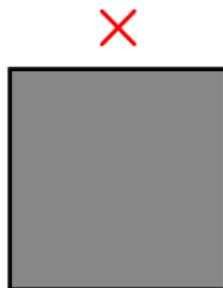
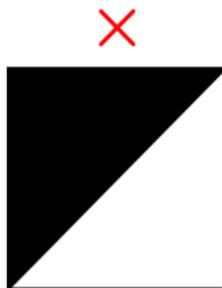
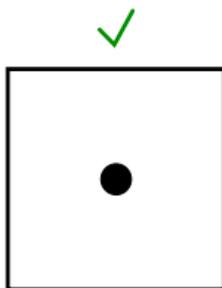
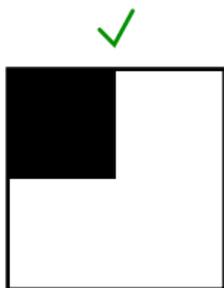
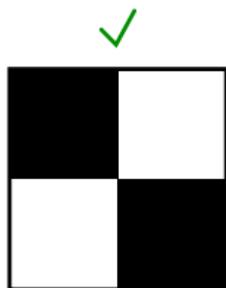
**Robustness:** occlusion, exposure, noise, does not require initialization

**Good localization:**

$$\sum_x \sum_y (\mathbf{I}[x, y] - \mathbf{I}[x + s, y + t])^2 > 0 \quad \forall (s, t) : \|(s, t)\| > 0$$

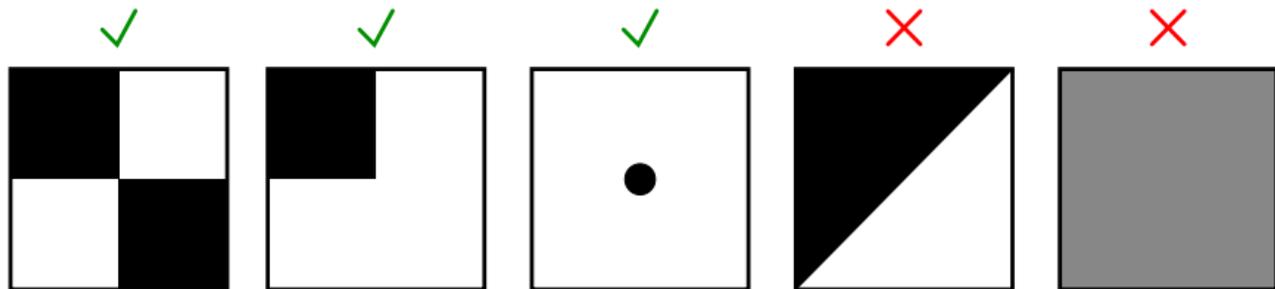
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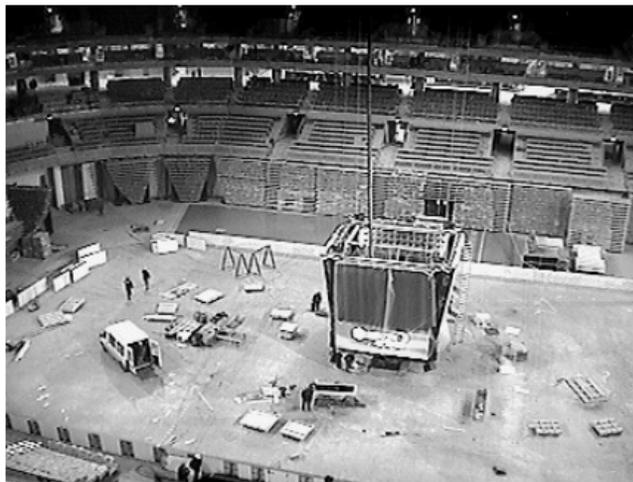


## Harris detector:

$$\mathbf{M} = \begin{pmatrix} \sum \left(\frac{\partial \mathbf{I}}{\partial x}\right)^2 & \sum \frac{\partial \mathbf{I}}{\partial x} \frac{\partial \mathbf{I}}{\partial y} \\ \sum \frac{\partial \mathbf{I}}{\partial x} \frac{\partial \mathbf{I}}{\partial y} & \sum \left(\frac{\partial \mathbf{I}}{\partial y}\right)^2 \end{pmatrix} \quad R = \det(\mathbf{M}) - \lambda (\text{trace}(\mathbf{M}))^2$$

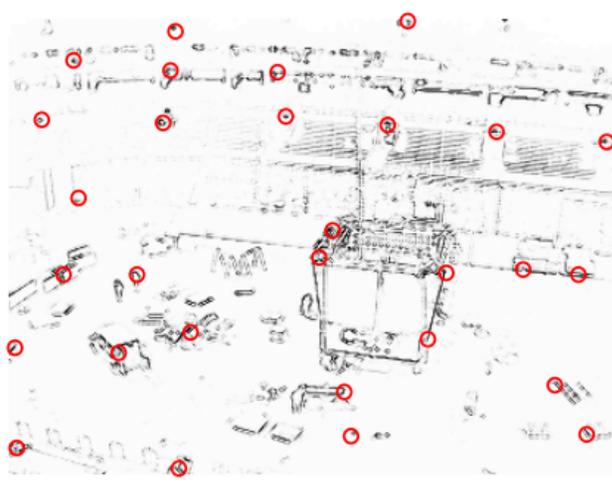
Repeat until specified number of points was found:

1. find pixel  $(x, y)$  where  $R[x, y]$  has maximal value
2. compute centroid  $(c_x, c_y)$  in a neighborhood of  $(x, y)$
3. store the centroid to the list of interesting points
4. remove point by drawing zero circle around  $(x, y)$  in  $R$

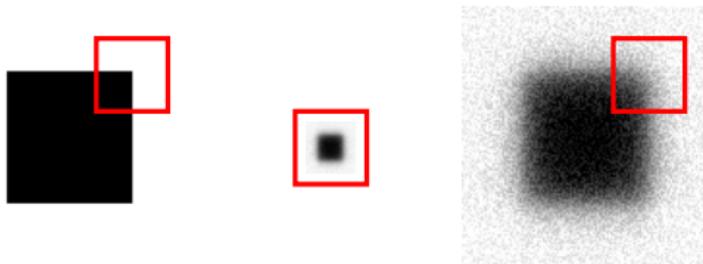


Repeat until specified number of points was found:

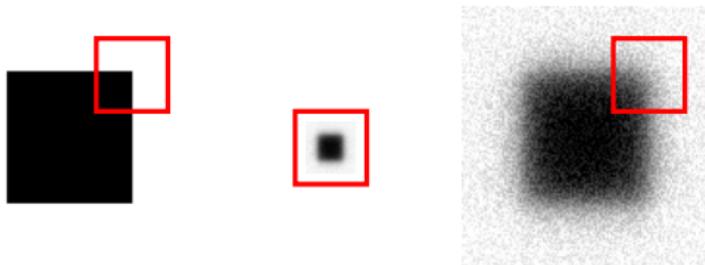
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**Problem:** output of Harris detector depends on the selected scale.



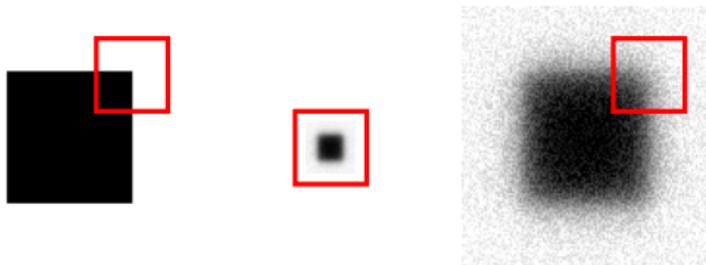
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**SIFT** (Scale-invariant feature transform):

Local extremes of normalized Laplacian-of-Gaussian scale-space.

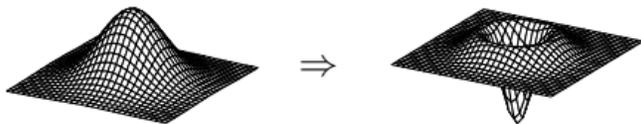
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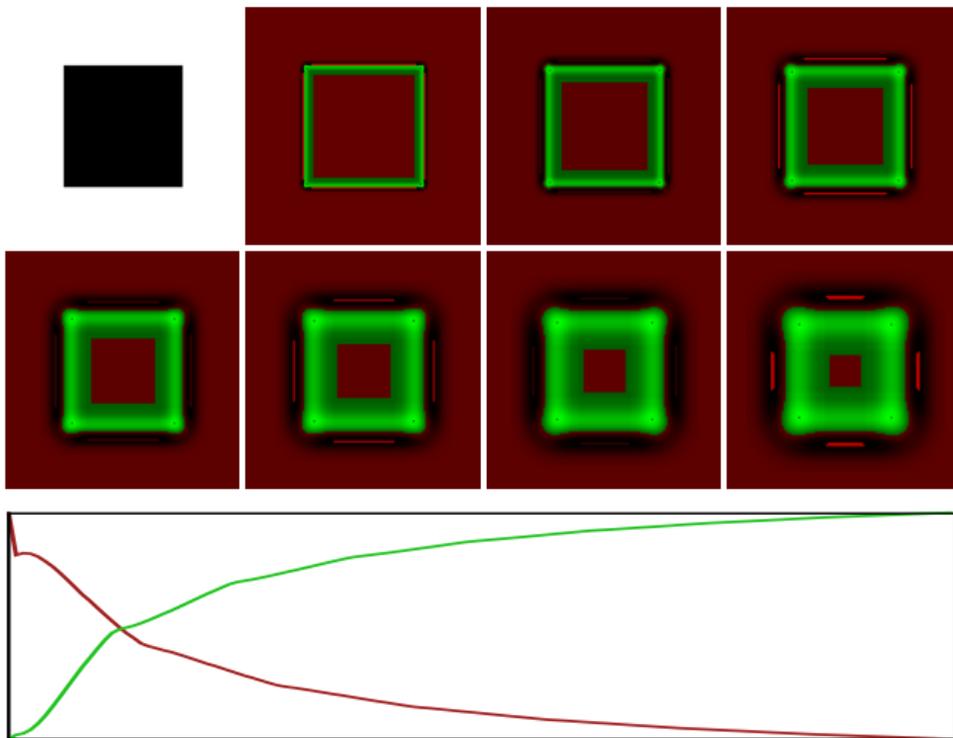
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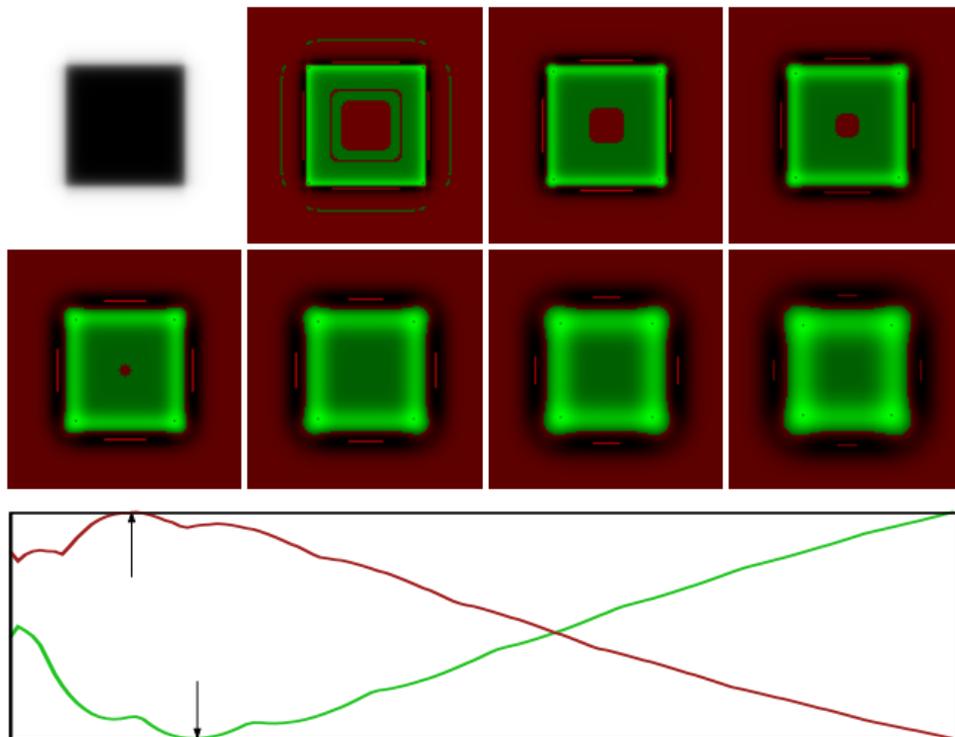
$$\mathbf{L} \circ \mathbf{G} = \nabla^2 \left( \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \right) = \frac{1}{\pi\sigma^4} \left( \frac{x^2+y^2}{2\sigma^2} - 1 \right) e^{-\frac{x^2+y^2}{2\sigma^2}}$$



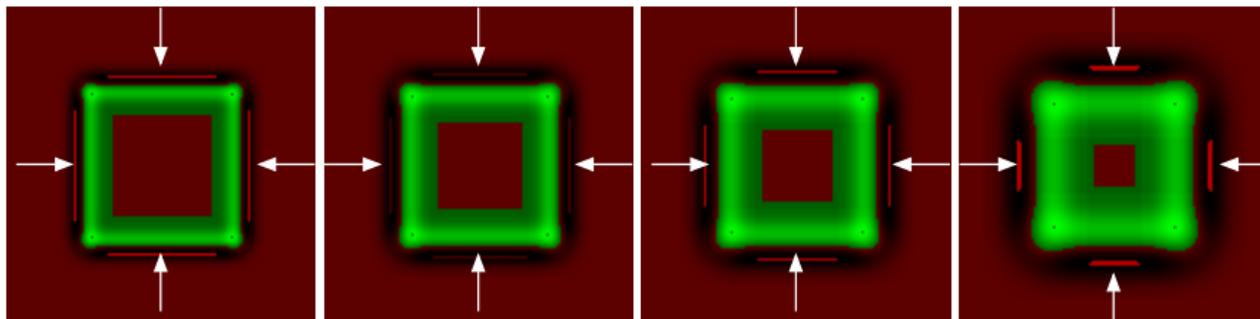
$L \circ G$  scale-space:



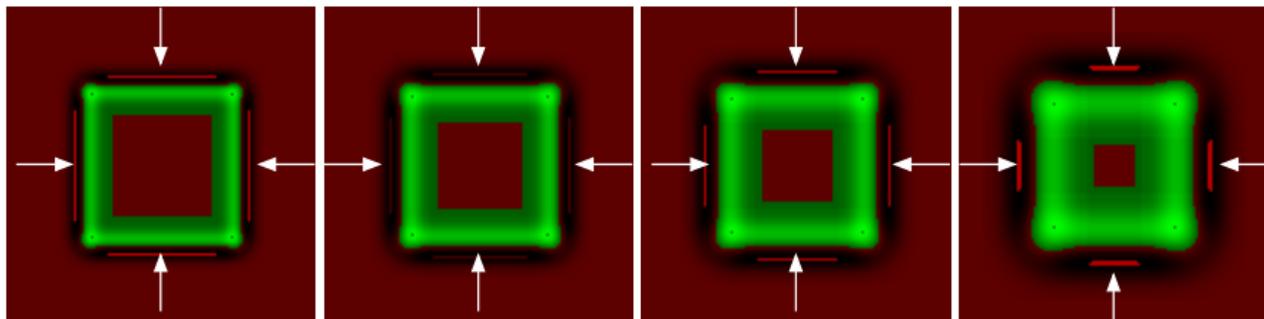
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**Problem:**  $L \circ G$  has strong response along edges.



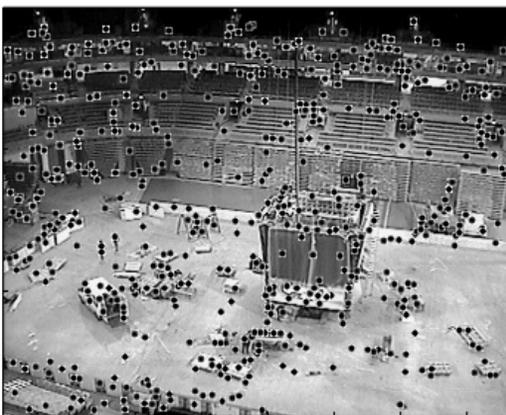
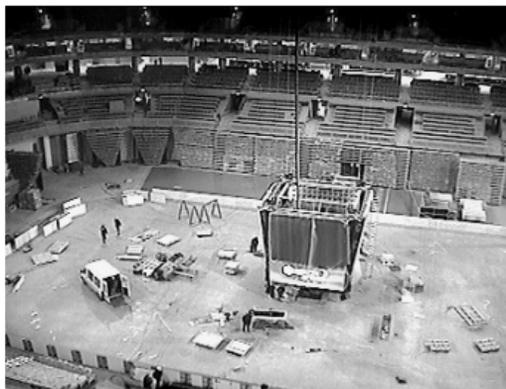
**Problem:**  $L \circ G$  has strong response along edges.



**Edge elimination:**

Large curvature across edge and small in perpendicular direction.

$$\mathbf{M} = \begin{pmatrix} \sum \left( \frac{\partial \mathbf{I}}{\partial x} \right)^2 & \sum \frac{\partial \mathbf{I}}{\partial x} \frac{\partial \mathbf{I}}{\partial y} \\ \sum \frac{\partial \mathbf{I}}{\partial x} \frac{\partial \mathbf{I}}{\partial y} & \sum \left( \frac{\partial \mathbf{I}}{\partial y} \right)^2 \end{pmatrix} \quad \frac{\text{trace}(\mathbf{M})^2}{\det(\mathbf{M})} < \frac{(r+1)^2}{r}$$



Normalized cross-correlation:

$$\frac{1}{\sigma_A \sigma_B} \sum_x \sum_y \left( A(x, y) - \hat{A} \right) \left( B(x, y) - \hat{B} \right)$$



Normalized cross-correlation:

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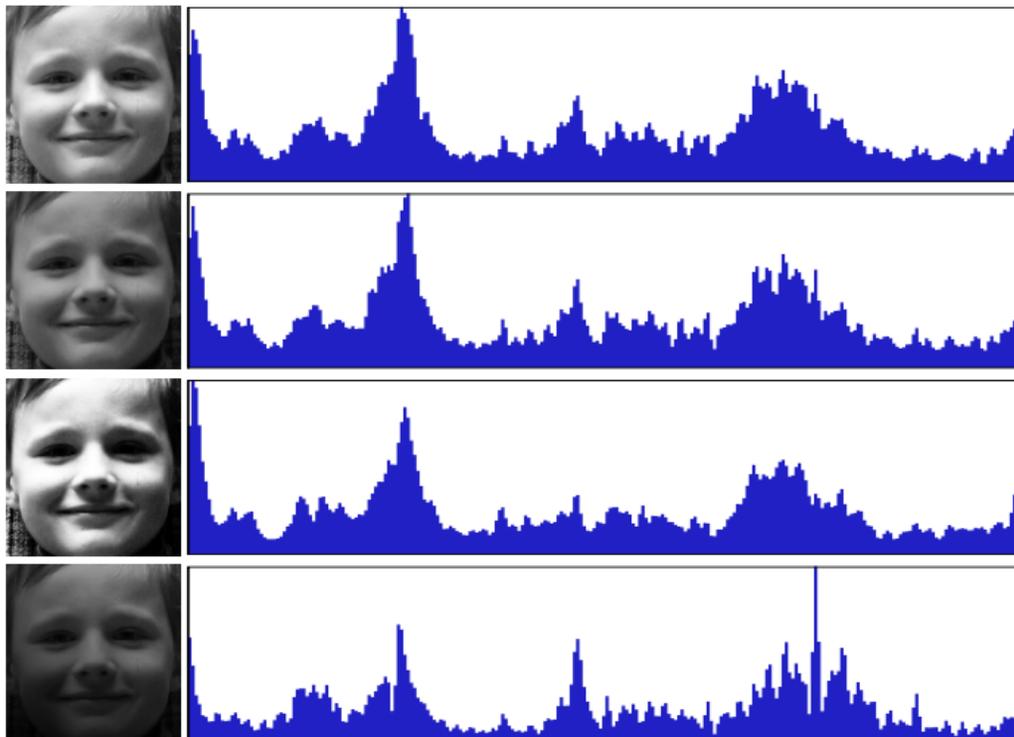


Histogram of gradient orientations (weighted by magnitude):

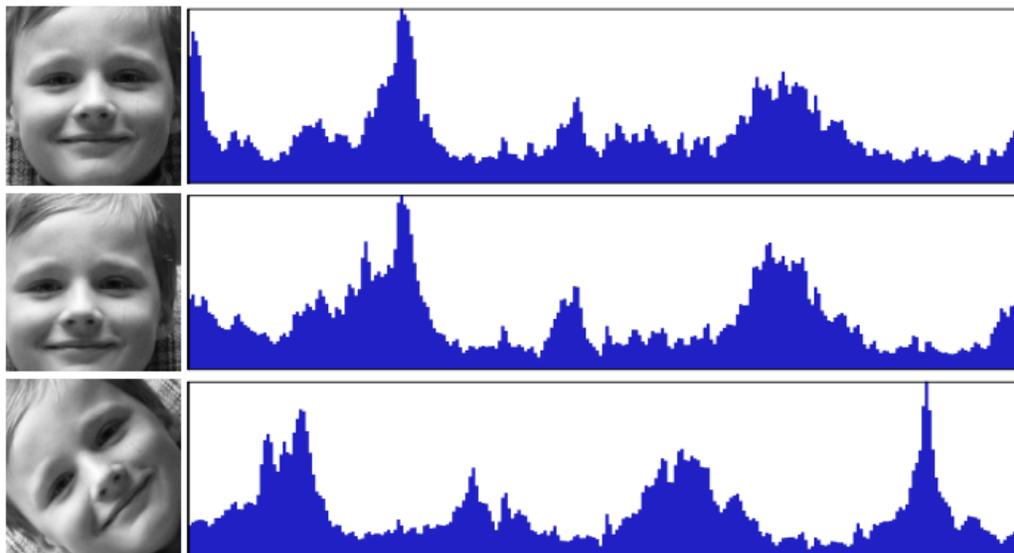
$$\alpha = \arctan \left( \frac{\mathbf{I} * \mathbf{G}'_y}{\mathbf{I} * \mathbf{G}'_x} \right) \quad w = \sqrt{(\mathbf{I} * \mathbf{G}'_x)^2 + (\mathbf{I} * \mathbf{G}'_y)^2}$$

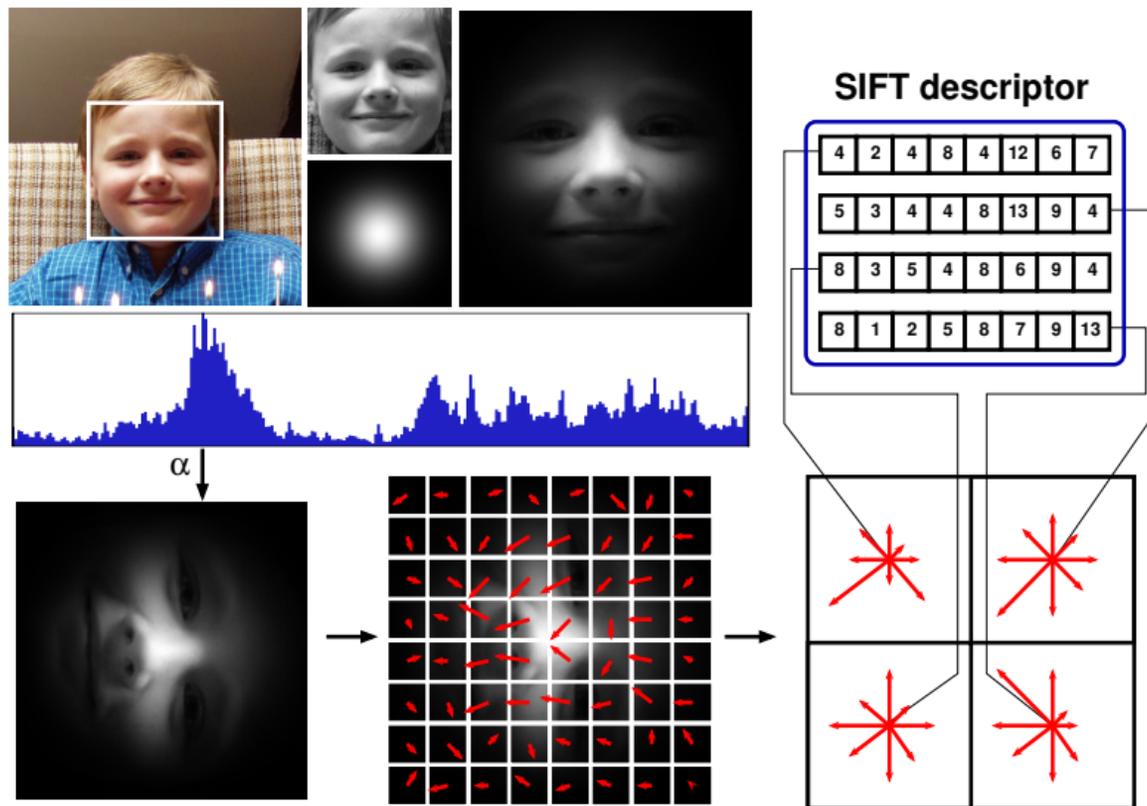


## Histogram of gradient orientations (illumination changes):



## Histogram of gradient orientations (rotation & translation):







1. Besides location also orientation and scale are important.
2. Single location may contain multiple interesting points.

Select  $\geq M$  best matching point correspondences:

$$x' = \frac{a_{11}x + a_{12}y + x_0}{f_1x + f_2y + 1} \quad y' = \frac{a_{21}x + a_{22}y + y_0}{f_1x + f_2y + 1}$$

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Built (overdetermined) linear system:

$$\begin{pmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x'_1x_1 & -x'_1y_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -y'_1x_1 & -y'_1y_1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 & -x'_2x_2 & -x'_2y_2 \\ 0 & 0 & 0 & x_2 & y_2 & 1 & -y'_2x_2 & -y'_2y_2 \\ \cdot & \cdot \\ \cdot & \cdot \\ x_M & y_M & 1 & 0 & 0 & 0 & -x'_Mx_M & -x'_My_M \\ 0 & 0 & 0 & x_M & y_M & 1 & -y'_Mx_M & -y'_My_M \end{pmatrix} \cdot \begin{pmatrix} a_{11} \\ a_{12} \\ x_0 \\ a_{21} \\ a_{22} \\ y_0 \\ f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ \cdot \\ \cdot \\ x'_M \\ y'_M \end{pmatrix}$$

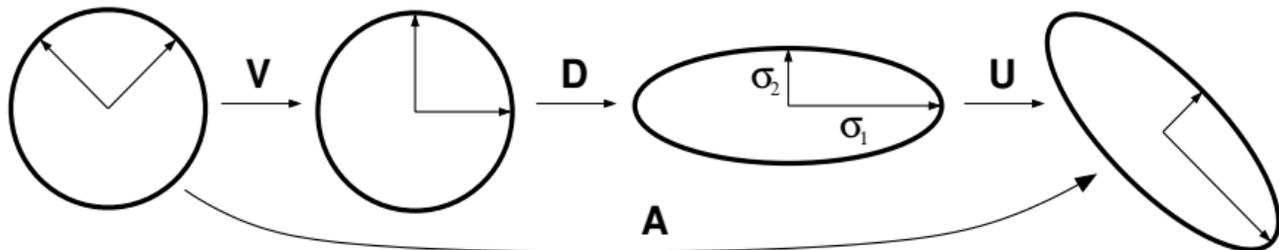
**4-point model:**  $M = 4$ , for  $M > 4$  use least-squares solution (SVD)

Each matrix  $\mathbf{A}_{M \times N}$  can be decomposed into:

$$\mathbf{A} = \mathbf{U} \cdot \mathbf{D} \cdot \mathbf{V}^T$$

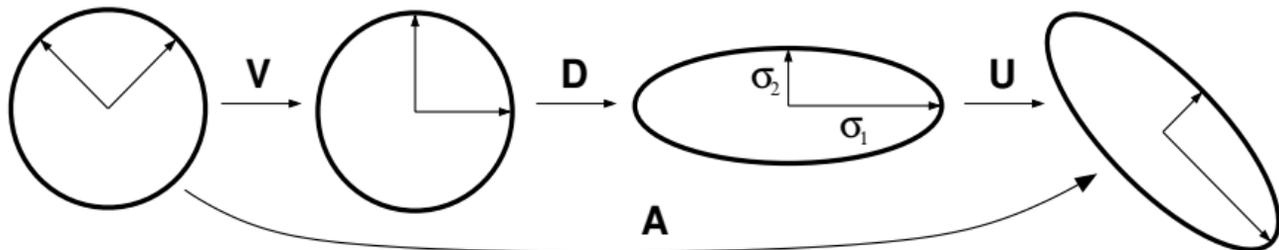
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$U$  &  $V$ :

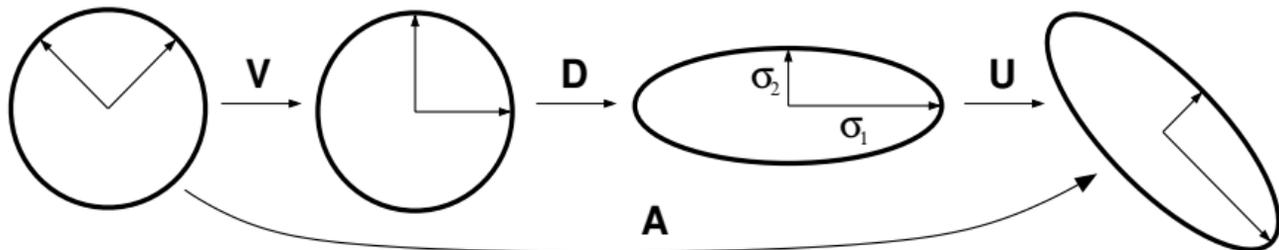
$$\|Ux\| = \|x\|$$

$$U^{-1} = U^T$$

$$UU^T = U^T U = I$$

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**U & V:**

$$\|Ux\| = \|x\|$$

$$U^{-1} = U^T$$

$$UU^T = U^T U = I$$

$$D_{M \times N} = \begin{pmatrix} \sigma_1 & 0 & \dots & 0 \\ 0 & \sigma_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_{\min(M,N)} \end{pmatrix}$$

$$\sigma_i = \sqrt{\lambda_k} \quad \lambda_k \Rightarrow \text{eigenvalue of } M = A^T A \quad Mx_k = \lambda_k x_k$$

$\mathbf{w} = \mathbf{v}_{\min(M,N)} \in \mathbf{V}$  minimizes  $\|\mathbf{A}\mathbf{w}\|^2$  subject to  $\|\mathbf{w}\| = 1$ .

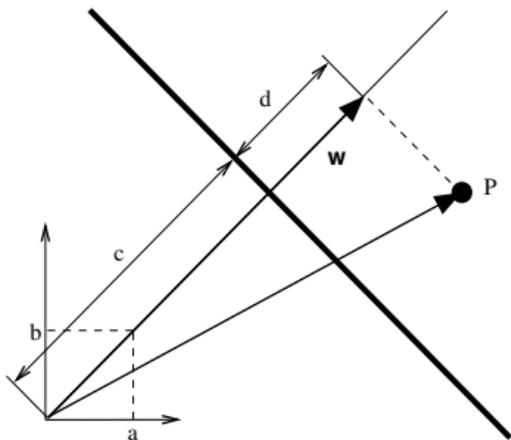
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Last column of  $\mathbf{V} \Rightarrow$  non-trivial solution to  $\mathbf{A}\mathbf{w} = 0$ .

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**Example:** Fit line  $\mathbf{w} \cdot \mathbf{x} + c$  to a set of points  $\mathbf{P}$ .



$$\mathbf{A} = \mathbf{P} - \mathbf{1}\hat{\mathbf{p}}^T \quad \hat{\mathbf{p}} = \frac{1}{n}\mathbf{P}^T\mathbf{1}$$

$$\mathbf{A} \Rightarrow \mathbf{U} \cdot \mathbf{D} \cdot \mathbf{V}^T \Rightarrow \mathbf{w} = \mathbf{v}_2$$

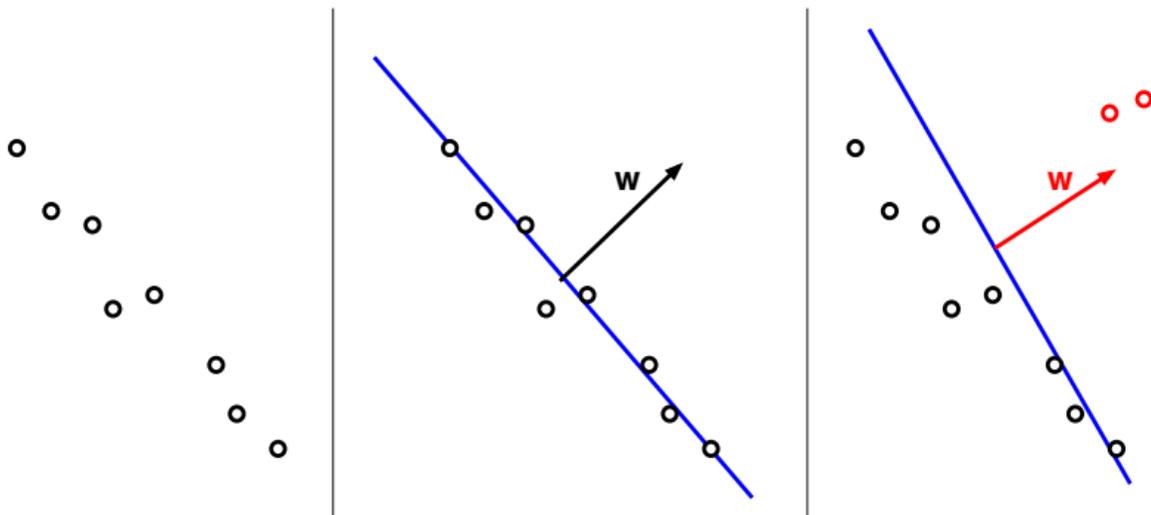
$$\mathbf{w} = (a, b)$$

$$\frac{\partial}{\partial c} \|\mathbf{P} \cdot \mathbf{w} - c \cdot \mathbf{1}\|^2 = 0$$

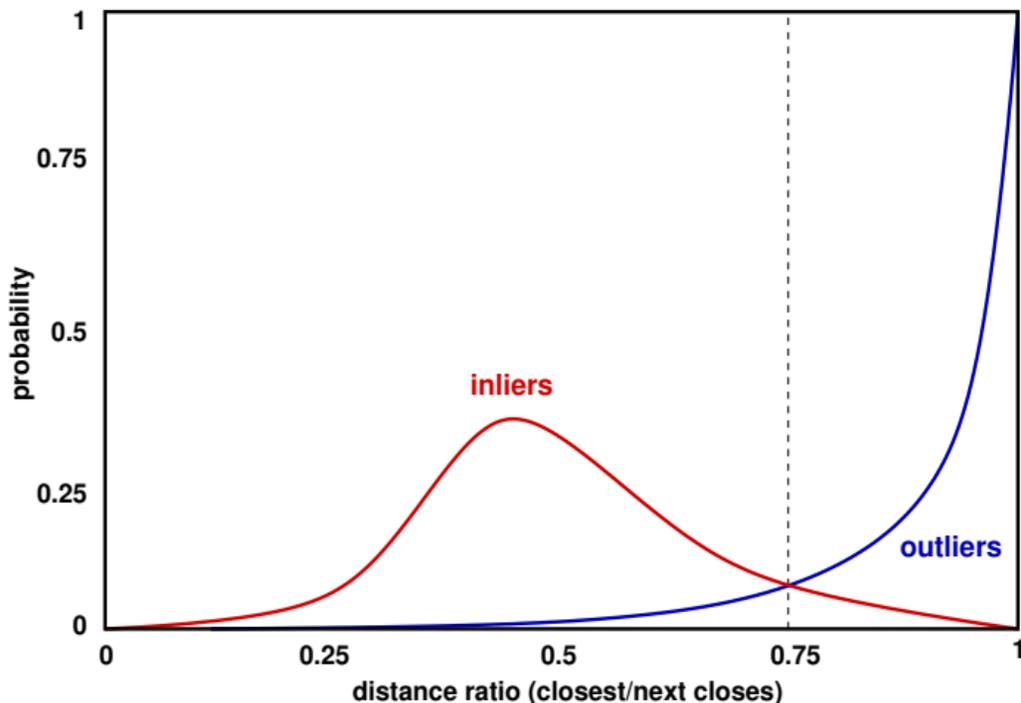
$$2(nc - \mathbf{w}^T \mathbf{P}^T \mathbf{1}) = 0$$

$$\Rightarrow c = \hat{\mathbf{p}}^T \mathbf{w}$$

**Problem:** outliers can affect accuracy of parameter estimation.

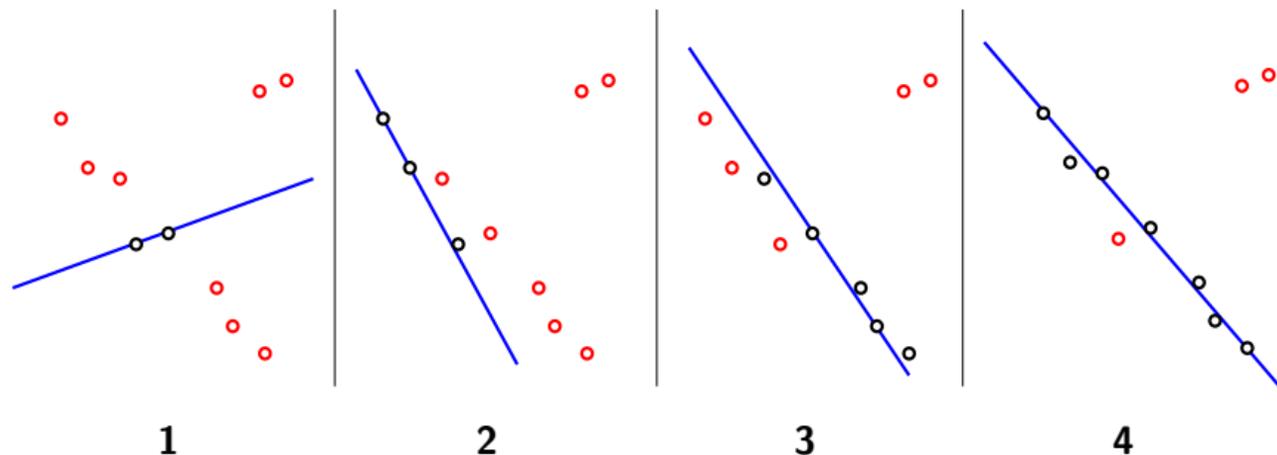


## Outlier detection:



## RANSAC (RANdom SAmple Consensus):

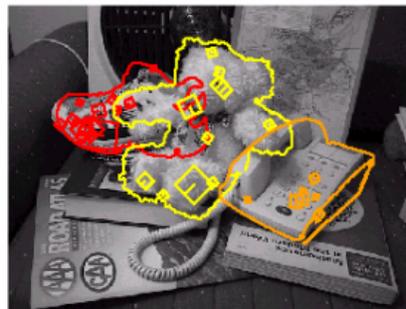
1. Select  $M$  random correspondences.
2. Compute exact model parameters  $\mathbf{p}$ .
3. Use  $\mathbf{p}$  to get number of outliers  $N$ .
4. If  $N < N_{\min}$  then  $N_{\min} = N$  and  $\mathbf{p}^* = \mathbf{p}$ .
5. If  $N_{\min} < \epsilon$  then return  $\mathbf{p}^*$  else goto 1.



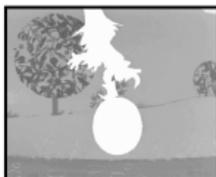
## Hyper-resolution panorama stitching:



## Image and object retrieval:



## Recovering background from occluded observations:



## Augmented reality:

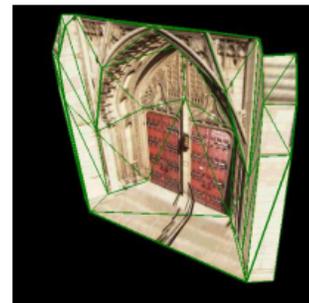
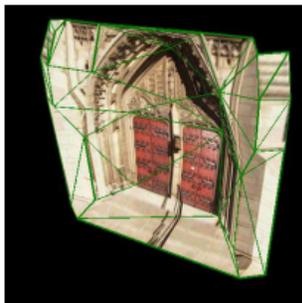
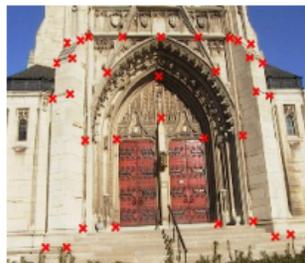
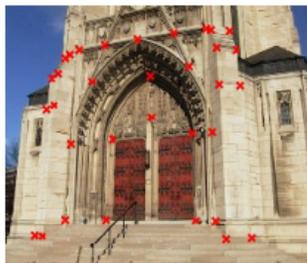


## Structure from motion:

source images



novel view synthesis



## Space carving:

